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A CATHODE RAY OSCILLOGRAPH FOR USE IN TOOL MAKING

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The cathode ray oscillograph is not only suitable for recording rapidly alternating electrical quantities, but it may also be used for the investigation of mechanical vibration phenomena. Since the frequencies of the mechanical phenomena to be investigated are usually considerably lower than those of electrical vibrations it was found desirable to develop a special oscillograph for this purpose, whose amplifier and time-axis deflection are so constructed that the frequency of a phenomenon, varying in any desired way, may fall to about 1 c/s., without the oscillogram being distorted to any serious extent. — In this article the cathode ray oscillograph GM 3 156 is described, which has been constructed on this basis. After a description of the amplifier and the time-axis generator several applications are dealt with.

An oscillograph is an instrument which serves to make visible or to record rapidly varying phenomena. Since electrical voltage variations must be supplied to the ordinary oscillograph, the latter was originally constructed almost exclusively from the point of view of measuring electrical phenomena. It was only gradually that auxiliary apparatus was developed to convert [1]) rapidly occurring mechanical phenomena into electrical phenomena, so that the oscillograph can now be used to an increasing extent in tool making also.

If one compares the applications of the oscillograph in tool making with those in electrotechnology, great differences will be found, especially with regard to the highest and the lowest frequencies of the phenomena to be investigated. In high-frequency technology one is often concerned with waves of a few metres in length, *i.e.* of vibration times of the order of 10^{-8} sec. Times of this order of magnitude also play a part in the non-periodic voltage and current changes which occur for example due to breakdown or to sparking upon breaking a contact [2]). In tool making, on the other hand, there is not the slightest necessity of reproducing such rapidly occurring phenomena. The mechanical vibrations of foundations, etc. are usually caused by rotating machines, and in such cases rotation frequencies of 3 000 r.p.m., *i.e.* 50 c/s. may already be considered high. If one is concerned with impact phenomena, the vibrations caused by the impact are determined by the mechanical resonance frequencies of the structural components, and these practically always lie within the acoustic region, *i.e.* not higher than a few thousand c/s.

Such a difference between electrical and mechanical problems is encountered at the lower limit of the frequency region which must be able to be covered in the reproduction. Alternating current technology generally works with a frequency of 50 c/s., lower frequencies seldom occur. In tool making, however, rotating engines with 120 r.p.m. are quite common, *i.e.* machines which show a fundamental frequency of 2 c/s. For the reproduction of these low frequencies the previously described cathode ray oscillograph (GM 3 152) is unsuitable, so that the need was felt of developing a separate oscillograph for tool making.

The frequency region in which the new cathode ray oscillograph, GM 3 156, produces an undistorted image is adapted to mechanical uses. The lowest fundamental frequency in this region amounts to 0.5 to 2 c/s. (30 to 120 cycles per minute) according to the nature of the phenomena to be recorded, while the upper limiting frequency amounts to about 10 000 c/s. In the case of the oscillograph GM 3152 previously described, which is intended primarily for electrotechnical application, the lower limiting frequency was 10 c/s and the upper 10^6 c/s; both limiting frequencies are therefore much higher in that case.

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[1]) On the subject of the conversion of mechanical voltages or vibrations into electrical voltages see Philips techn. Rev. **5**, 26 and 241, 1940.

[2]) The electron beam of a cathode ray oscillograph is capable of following these rapid phenomena, but this is not true of the electrical apparatus of the oscillograph. In Philips techn. Rev. **4**, 198, 1939 it is explained in detail what measures are taken in the amplifier of the cathode ray oscillograph GM 3 152 to obtain an undistorted reproduction up to as high frequencies as possible.

In this article we shall discuss the results of the downward shift of the effective frequency region on the construction of the oscillograph. Several examples of applications will then be dealt with.

The amplifier

In order to make visible on the screen of a cathode ray tube the very small A.C. voltages which are obtained upon the conversion of alternating mechanical quantities into electrical voltages, an amplifier with two stages is used. The input voltage of the second stage of this amplifier comes from the anode current alternations of the first stage. When an attempt is made to adapt such an amplifier to the amplification of very low frequencies, the difficulty is encountered that the slow, but sometimes quite considerable variations in the anode current, which are caused by fluctuations in the mains voltage when the amplifier is fed from the mains, are also transmitted to the following stage and lead to a distortion of the oscillogram. This difficulty can, however, be overcome by constructing the whole amplifier on the push-pull principle.

The application of the push-pull principle was of course obvious, since, as has previously been explained, the vertical deflection of the cathode ray is best brought about with a balanced voltage. For this reason even in the case of the amplifier of the oscillograph previously described the last stage was in push-pull connection. The necessity of limiting the influence of mains voltage variations has, however, led to the application of the push-pull principle in the first stage of the amplifier also.

In *fig. 1* the connections employed are shown. With the help of several variable resistances the input signal of the valve B_1 coming from the externally applied A.C. voltage, can be changed continuously and in steps. The alternating anode current of this valve passes through the resistance R_k and thereby causes such a variation of the cathode potential that the voltage difference between grid and cathode of the valve becomes smaller (inverse feed-back). At the same time an input voltage with opposite sign occurs between grid and cathode of the valves B'. In this way therefore a balanced voltage is obtained from the very beginning. The A.C. voltages in opposite phases over the anode resistances R_a are supplied *via* the condensers C to the second stage of the push-pull amplifier and again amplified. The output A.C. voltage of the second stage is fed *via* two coupling condensers to the first set of plates for vertical deflection of the cathode ray tube.

If a fluctuation of the mains voltage occurs, the

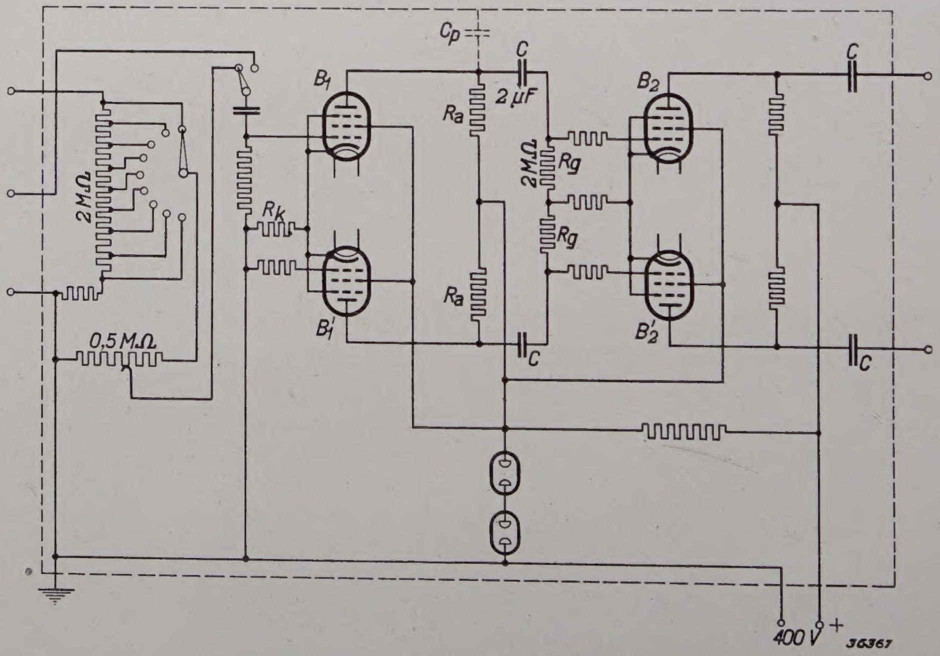

Fig. 1. Connections of the amplifier for the cathode ray oscillograph GM 3 156. It consists of two stages in push-pull connection and in the frequency region between 0.1 and 10 000 c/s it gives a voltage amplification by a factor 10 000.

anode voltages of the valves B_1, B'_1 and B_2, B'_2 will change in equal phases and not in opposite phases as takes place upon application of an input signal. Such a voltage fluctuation thus finally has the result that both deflection plates undergo the same change of potential, so that the difference of potential remains the same. The deflecting force on the electron beam is therefore unaltered; the beam merely undergoes an additional acceleration of retardation, which of course also results in a shift, although much smaller, of the oscillogram. It is therefore desirable to limit the voltage fluctuations themselves at the same time. This could be done with the help of two neon lamps in series as stabilizers, as indicated in the diagram. The result of these measures is that a sudden variation in the mains voltage of one per cent in the most unfavourable case causes a shift of the spot on the screen of 1.5 mm, and a slow change of the mains voltage has no observable results.

The reproduction characteristic of an amplifier stage at low and high frequencies is mainly determined by the anode resistance R_a, the gridresistance R_g, the capacity of the coupling condenser C and the capacity C_p between the coupling elements and earth. In the article already referred to several times about the cathode ray oscillograph GM 3152 it was explained that the lower limit f_1 of the frequency region within which the amplification is satisfactorily constant is determined by the condition

$$2\,\pi f_1\, R_g\, C = 1, \quad . \; . \; . \; . \; . \quad (1)$$

while the upper limit is determined by [3]):

$$2\,\pi f_2\, R_a\, C_p = 1 \quad . \; . \; . \; . \; . \quad (2)$$

In the oscillograph for use in tool making $R_g = 2$ MΩ and $C = 2$ μF. Thus according to equation (1)

$$f_1 = \frac{1}{2\pi \cdot 2 \cdot 2} = 0.04 \text{ c/s}.$$

The length of the cycle is thus 25 seconds. The definition of f_1 in equation (1) is such that the amplification for this limiting frequency with the connections here used would only amount to 35 per cent of its value in the flat part of the characteristics. If higher requirements are made with respect to lack of deformation in the reproduction, for instance if it is required to know the frequency region in which the amplification does not lie more than 10 per cent below the maximum value, then instead of equation (1), one obtains the following:

$$2\pi f_1\, R_g\, C = 4 \quad . \; . \; . \; . \; . \; . \; . \quad (1a)$$

and one calculates a lower limit of 0.17 c/s (10 c/min.)

In the case of the previously described oscillograph the lower limiting frequency for a faithful reproduction was considerably higher, because the coupling condensers C were much smaller. An enlargement of the coupling condensers was undesired in that case in connection with the reproduction of high frequencies. The coupling condensers contribute considerably to the capacity C_p, which according to equation (2) determines the upper limit of the flat part of the frequency characteristic. In the oscillograph under consideration a large value of this capacity is no objection, since the upper limiting frequency is chosen much lower. The product R_aC_p in equation (2) may be larger by a factor 100 than in the oscillograph GM 3 152, and thus in spite of the larger value of C_p, the anode resistance R_a could also be made larger, whereby the amplification in the flat part of the characteristic could be increased by a factor 6. In *fig. 2* the variation of the amplification is given as a function of the frequency.

It might be supposed that the extension of the region of constant amplification to lower frequencies has been carried too far. A signal which alternates with a frequency of only a few cycles per second can easily be recorded with registering pointer instruments so that there is no need of an oscillograph for this purpose. It is found, however, that an undistorted reproduction at these low frequencies is only possible for sinusoidal vibrations, while in the case of non-sinusoidal vibrations deviations already begin to occur at considerably higher frequencies. We shall demonstrate this by calculation of the amplification of a signal of a rectangular form.

As connections for the amplifier we shall choose the simplified scheme according to *fig. 3*. If an A.C. voltage of any given form $V(t)$ is applied *via* the condenser C to the grid leakage resistance R of the first amplifier valve, then one finds for the voltage V_1 over this resistance the following differential equation:

$$\frac{\mathrm{d}V_1}{\mathrm{d}t} + \frac{V_1}{RC} = \frac{\mathrm{d}V}{\mathrm{d}t}.$$

The general solution is

$$V_1 = \left(V_0 + \int_0^t \frac{\mathrm{d}V}{\mathrm{d}t}\, e^{t/RC}\, \mathrm{d}t\right) e^{-t/RC} \quad . \quad (3)$$

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[3]) The amplification of a stage is practically constant over the greatest part of the region between these limits, and at the limits themselves it falls to $\sqrt{1/2}$ (71%) of its constant value.

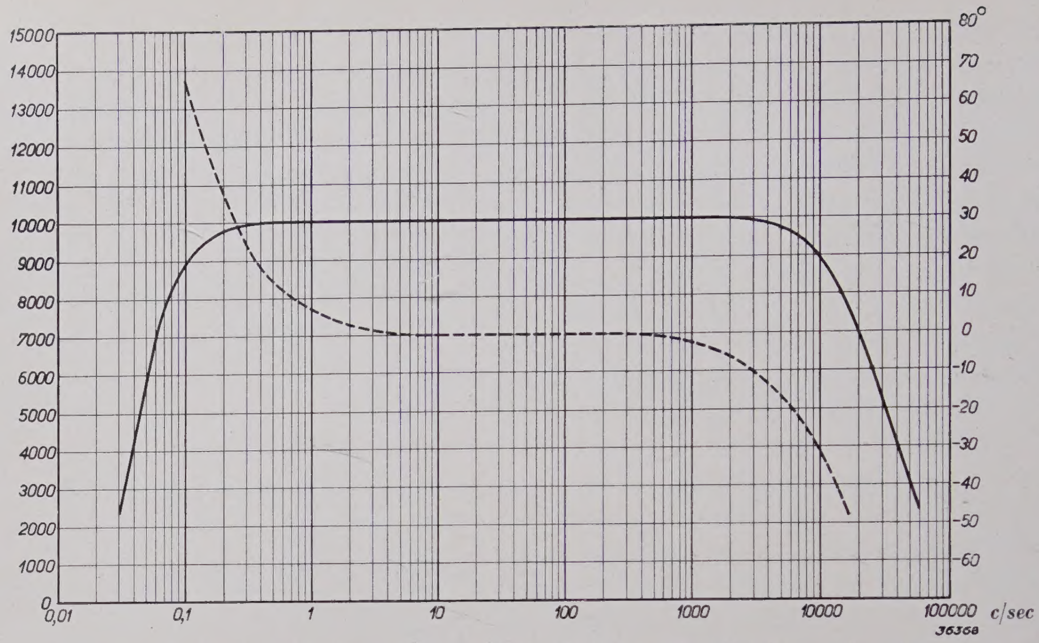

Fig. 2. Frequency characteristic of the amplifier. Full line, amplification, broken line, phase shift, as functions of the frequency. If a decrease in the amplification of 10% is allowed, the useful reproduction range extends from 0.1 to 10 000 c/s. If a phase shift of 5° is allowed, which, as far as the possible distortion af a given oscillogram is concerned, corresponds approximately to the decrease of 10% mentioned, the amplifier can only be considered useable in the frequency range from 1.5 to 1 500 c/s.

Let us assume that the signal voltage $V(t)$ is equal to zero until the moment $t = 0$, then suddenly assumes a definite value V_t and is later constant for some time. From equation (3) it then follows that

$$V = V_0\, e^{-t/RC} \quad . \; . \; . \; . \; . \; . \; . \quad (4)$$

The output voltage of the amplifier valve amounts to $A V_1(t)$, where A is the amplification of the first stage. This voltage has exactly the same significance for the second stage as the voltage $V(t)$ for the first stage, so that for the control grid voltage of the second valve we may immediately write

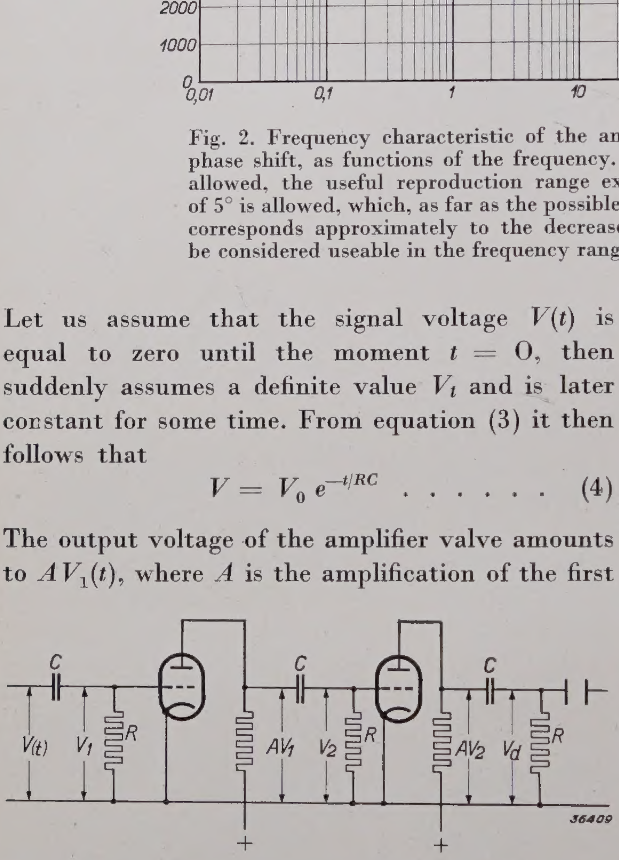

Fig. 3. Simplified diagram of the amplifier.

$$V_2(t) = \left(A V_0 + \int_0^t A \frac{\mathrm{d} V_1}{\mathrm{d}t}\, e^{-t/RC} \mathrm{d}t \right) e^{-t/RC},$$

where V_1 must be substituted according to equation (4). When the integration is performed one obtains:

$$V_2(t) = A V_0 \left(1 - \frac{t}{RC} \right) e^{-t/RC} \quad . \; . \; . \quad (5)$$

This voltage is again amplified by a factor A in the second stage, and *via* a condenser C again applied to a resistance R. The final deflection voltage V_d can therefore again be calculated according to equation (3), where V must be set equal to $A V_2$. One then obtains

$$V_d\,(t) = A^2 V_0 \left(1 - \frac{2t}{RC} + \frac{t^2}{2\, R^2\, C^2} \right) e^{-t/RC}. \quad (6)$$

It is thus clear that the voltage surge V_0 amplified at the first moment by a factor A^2 is transmitted to the deflection plates. Instead of remaining constant, however, the voltage then decreases exponentially. (An oscillation occurs thereby, as shown in *fig. 4* in which the calculated variation of the voltage is given.) If we consider relatively

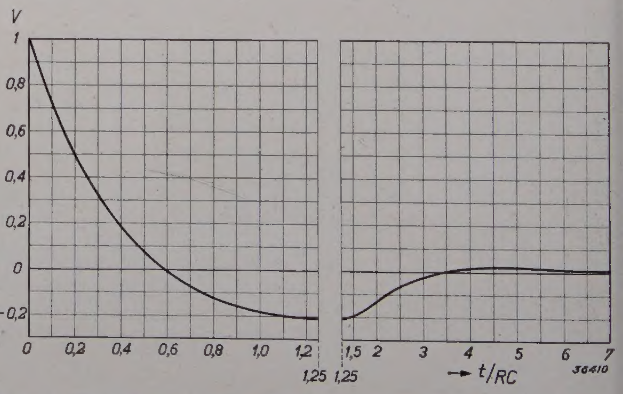

Fig. 4. Variation of the voltage on the deflection plates as a function of the time upon a commutation surge, see equation (6).

short times ($t \ll RC$) we may approximate the variation of the voltage by taking the first term of the series of e functions, and we obtain

$$V_d \approx A^2 V_0 \left(1 - \frac{3t}{RC}\right).$$

For faithful reproduction the requirement must be made that this shall be a fairly constant voltage. The corresponding input voltage is constant in any case. If V_d is allowed to decrease 10 per cent the maximum value for the time t becomes

$$t_m = \frac{1}{30} RC .$$

The input voltage may not remain constant longer than t_m if non-permissible distortion is not to occur. If RC is expressed by the lower limiting frequency according to equation (la) one finds

$$t_m = \frac{1}{15\pi f_1} = \frac{1}{47 f_1}.$$

The maximum duration of a surge upon switching on which can be reproduced with satisfactory reliability is therefore 47 times shorter than the maximum duration of a cycle of a sinusoidal curve which is reproduced with the same accuracy. For a rectangular A.C. voltage which may be considered as a succession of such starting surges, the same considerations are valid in principle as for one such surge; the duration of one period of the square sine then amounts to $2t_m$. Good reproduction of sinusoidal vibrations to a lower limit of 0.17 c/s thus corresponds to a good reproduction of rectangular vibrations ("square sines") to a lower limit of $47 \times 0.17/2 = 3.75$ c/s. It is thus clear that upon consideration of non-sinusoidal vibrations the region of undistorted reproduction is by no means carried too far toward low frequencies. We shall see in the following oscillograms how this is manifested in practice.

In *fig*. 5 three oscillograms of square sines are reproduced. Diagram *a* was obtained by applying the voltage which had a frequency of 2 c/s directly to the deflection plates of the cathode-ray tube. Diagram *b* shows that this shape can still be reproduced by the amplifier without too great distortion. Finally it may be seen from diagram *c* that this is no longer the case at appreciably lower frequencies, at a frequency of 0.5 c/s, instead of the horizontal sections, the oscillogram shows exponentially falling lines, thus a strong distortion, although no noticeable decrease in the amplification yet occurs at this frequency.

The fact that the square sine of low frequency is so much more badly reproduced than the pure sine, seems somewhat paradoxical; the square sine may be considered as composed of a pure sine and higher harmonics, and when the component with the lowest frequency is not yet appreciably weakened, this is decidedly not the case with the higher harmonics. It must, however, be kept in mind that, due to the amplification of a composite vibration, not only can the relative amplitudes of the harmonics change, but also the relative phases, and the latter effect already appears at considerably higher frequencies than the former, as may be seen from the broken-line curve in fig. 2. By the disturbance of the relative phases the sum of two vibrations in the output signal may take on a quite different form than that possessed by the original vibration, even though the relation between the amplitudes is not changed by the amplification.

In addition to distortions due to an amplification which varies with the frequency and distortions due to a phase shift which varies with the frequency, an amplifier in general also possesses a so-called non-linear deformation, which means that a sinusoidal input signal does not lead to an exactly sinusoidal output signal. The deviation from the sine shape increases with the amplitude of the

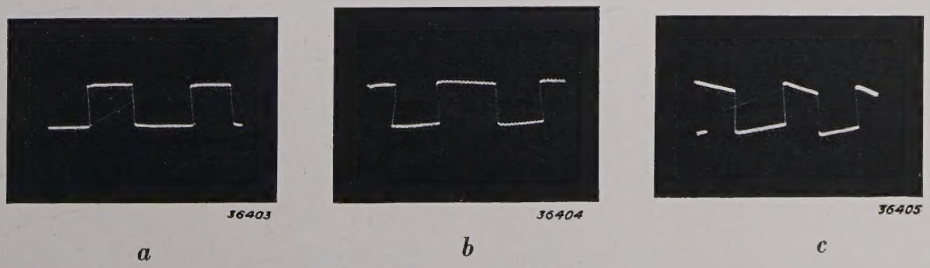

Fig. 5. Oscillograms of a rectangularly alternating voltage.
a) The voltage in question (square sine of 2 c/s) is applied directly to the plates for vertical deflection of the cathode ray tube.
b) The same voltage is fed to the cathode ray tube *via* the amplifier. Practically the same diagram results.
c) The frequency of the square sine is decreased to 0.5 c/s. A distortion of the oscillogram can now be clearly observed.

input signal, and with the amplifier of the cathode ray oscillograph in question it amounts to about 2 per cent in the least favourable case, *i.e.* with a signal corresponding to an image height of 6 cm. This deviation can scarcely be observed on the screen of the tube, and may therefore be considered permissible.

The time axis generator

The function of the time axis generator is to supply a voltage which is proportional to the time, and which suddenly falls to its initial value after reaching a given maximum, so that a saw-tooth form is obtained (*fig. 6*). If this voltage is applied to the plates for horizontal deflection of the cathode ray tube, and the voltage of the phenomenon to be recorded is simultaneously applied to the plates for vertical deflection, one sees on the screen of the tube a diagram which represents the quantity to be recorded as a function of the time.

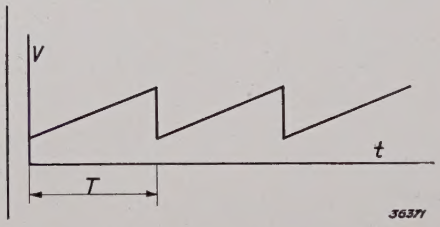

Fig. 6. Variation of the horizontal deflecting voltage, generated by the time axis generator, as a function of the time. At an amplitude of the horizontal deflection of 8 cm the "sawtooth" period T can be varied from 0.005 sec to 4 sec.

The velocity of the sawtooth motion must be able to be adapted to the frequency of the phenomenon to be recorded. This means that the region within which the frequency of the sawtooth motion can be varied must also be chosen considerably lower with the oscillograph for use in tool making than in the cathode ray oscillograph GM 3 152. The periodic falling back of the voltage can in this case be realized very easily by means of gas triodes, which could not be used in the previously described oscillographs, because they have too much time lag for the highest time axis frequencies of more than 100 000 c/s occurring in those oscillographs. For the maximum time axis frequencies of the cathode ray oscillograph for use in tool making, however, gas triodes may very well be used. Moreover, the circuit of the sawtooth generator obtained with gas triodes offers great advantages when it is desired to pass over to a much lower limiting frequency than is the case with the existing oscillographs.

In *fig. 7* the connections used are given. The pentode L_p charges the condenser C with a constant current, and the cathode potential of the gas triode L_g decreases proportionally to the time. Between the cathode and the anode of the gas triode a potential difference thus occurs, which will lead to breakdown as soon as it exceeds a certain value. This value is mainly determined by the bias of the grid g. The breakdown occurs approximately when the potential of the cathode has fallen below that of the grid. Due to the breakdown the condenser C is discharged with a high current, so that the cathode potential again rapidly increases to its initial value; the discharge of the gas triode is hereby extinguished and the process begins anew.

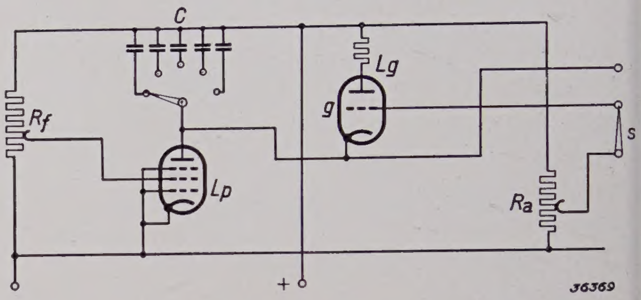

Fig. 7. Connections of the time axis generator. L_p pentode, L_g gas triode. By changing the capacity C and adjusting the potentiometer R_f the sawtooth frequency is regulated; with the help of the potentiometer R_a the sawtooth amplitude is regulated.

By commutation of the condenser C the frequency of the time axis voltage can be affected in steps within wide limits, while a continuous regulation is possible by changing the screen grid voltage of the pentode L_p with the help of the potentiometer R_f, and regulating in this way the charging current of the condenser C. The amplitude of the sawtooth motion is adjusted with the potentiometer R_a from which the D.C. voltage for the grid of the gas triode is taken. Finally it is possible to synchronize the period of the sawtooth voltage with the help of an externally applied voltage, which is applied instead of the short circuit to the terminals s, *i.e.* to the grid of the gas triode. For this purpose the voltage of the phenomenon to be investigated may itself be used and this is taken from the output of the amplifier.

If it is desired to record non-periodic phenomena, such as those occurring upon throwing a switch, instead of the oscillating sawtooth deflection it is in many cases advantageous to use a single non-periodic linear horizontal deflection of the fluorescent spot which begins at a suitable moment. This can be obtained by applying to the grid of the gas triode such a high voltage before the beginning of the linear motion, that the discharge is never extinguished and the condenser C remains practically uncharged. At the moment when the phenomenon to be investigated begins, the normal voltage is applied to the grid of the gas triode with a suitable synchronizing arrangement, a mechanically driven contact, for instance. At that moment the discharge is extinguished and the fluorescent spot begins its linear motion. After having covered a certain distance the spot will jump back to the initial position and begin its motion again. The mechanically driven contact will therefore preferably be so arranged that before this first period of the sawtooth has elapsed the grid is again brought to a high voltage, so that the spot jumps back and then remains in its resting position.

By moving the sliding contact on the resistance R_f and commutation of the condenser C the velocity of the linear motion of the spot can be regulated continuously or in steps between 2 cm/sec and 16 000 cm/s. For the sawtooth motion this means a variation of the time axis frequency between 1/4 and 2 000 c/s for a length of the time base of 8 cm.

The cathode ray tube

The oscillograph for use in tool making is provided with a cathode ray tube DN 9-3 or with the post-acceleration tube DN 9-5 which was described in detail in the previous issue of this periodical 4). The cathode of the tube is at a potential which is 1 000 volts negative with respect to earth, while the space between the deflection plates is at about earth potential, so that a voltage to be investigated with respect to earth can also be connected directly to the plates of the cathode ray tube, *i.e.* without the use of the amplifier. The electrons which have passed the deflecting systems can be accelerated a second time when the post-acceleration tube is used, as was explained in the article referred to, by means of a positive voltage on the post-acceleration electrode. For this purpose there is a special supply apparatus (GM 4 198) which can give an adjustable high D.C. voltage with respect to earth (see *fig. 8*). The highest voltage which may be applied to the post-acceleration electrode amounts to 4 000 volts with respect to earth (5 000 volts with respect to the cathode).

Due to the stronger acceleration of the electrons the picture is much brighter, so that photographic recording of the oscillogram even with simple cameras with relatively small relative aperture is possible, while at the same time there is the possibility of projecting the image and thereby making it visible to more persons at one time. For this purpose there is an auxiliary projection apparatus (GM 4 198) with a lens of 50 mm diameter and a focal length of 150 mm. The distance from the projection screen may be 1 to 5 m, and an enlargement by a factor 6 to 30 is thereby obtained.

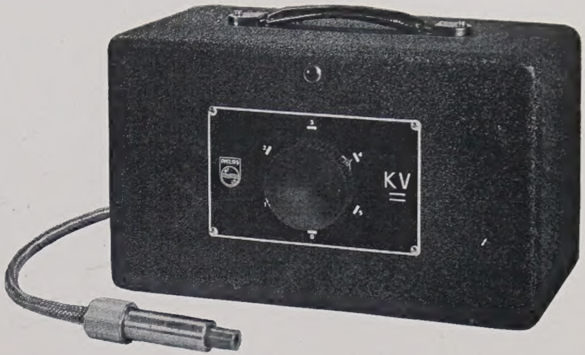

Fig. 8. The 5 000-volt supply apparatus GM 4 198 for the post-acceleration tubes of the cathode ray oscillograph GM 3 156. The apparatus contains a high-voltage transformer, a rectifier valve and a filter for smoothing the D.C. voltage obtained. As may be seen it can be set at five different voltages with the help of a switch.

Several applications

The oscillograph for use in tool making will be chiefly employed to investigate vibration phenomena and rapidly alternating tensions in parts of machines. Since these subjects have been dealt with in detail previously 5) we shall here only discuss a few less obvious applications.

Measurements on rotating components

With the help of a coil and a permanent magnetic core the roundness of shafts and flywheels can be tested in a very simple way. In *fig. 9* an arrangement used for this purpose is shown schematically, while *fig. 10* is a photograph of the arrangement. The iron shaft A turns at a distance of 1 mm from the magnet M with the coil S. If the air gap L

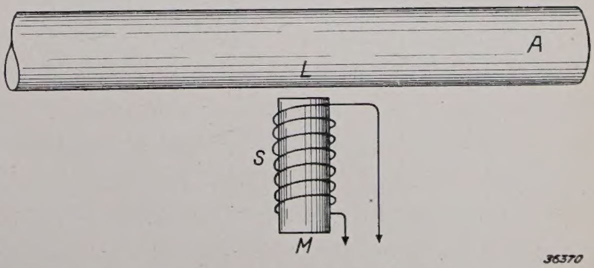

Fig. 9. Arrangement for the measurement of the lack of roundness and excentricity of shafts. A shaft, L air gap, S coil, M permanent magnet. The variation of the width of the air gap due to lack of roundness and excentricity excites an alternating voltage in the coil.

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4) A cathode ray oscillograph with post-acceleration Philips techn. Rev. 5, 257, 1940.

5) See the articles referred to in footnote 1).

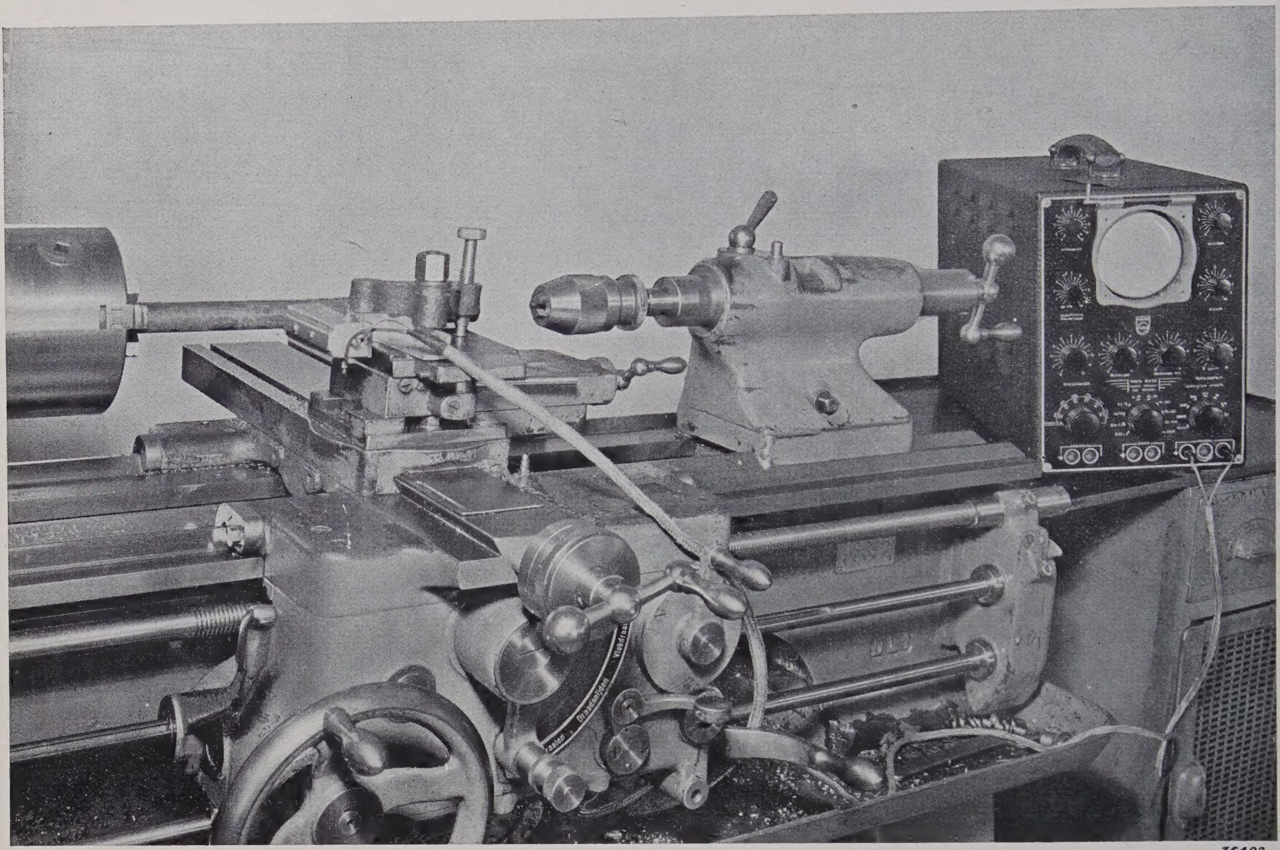
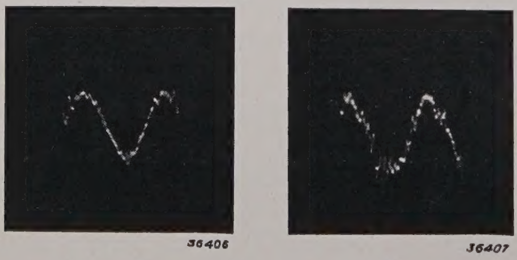


Fig. 10. Arrangement for the measurement of the lack of roundness of shafts.

between the shaft and the magnet varies in width, a voltage will be induced in the coil *S* which is proportional to the velocity of the change in width. When the shaft is perfectly round and does not vibrate as it turns, the induced voltage will be zero. The oscillogram thus indicates by its shape whether there is a lack of roundness, excentricity or mechanical vibration.



a *b*

Fig. 11. Two oscillograms obtained with an arrangement according to fig. 9. The sinusoidal vibration indicates an excentricity, while the indentations which are deeper in case *b* than in *a* are caused by irregularities of the surface.

Fig. 11 gives an example recorded with a shaft with a speed of 1000 r.p.m. and a diameter of 50 mm. In *a* we can distinguish voltage variations caused by slight irregularities and an excentricity. In *b* the same may be seen for a shaft with a rougher surface.

The frequency of the vibration which is caused by the excentricity amounts in this case to 16 2/3 c/s, so that the previously described oscillograph which gives a practically constant amplification to a lower limit of 10 c/s would also be able to indicate this excentricity practically undistorted. The use of the oscillograph specially constructed for tool making has the advantage, however, that the phase of the excentricity is also given quite correctly, since the phase deviation above 4 c/s is less than 1°.

Phase measurements at low frequencies are not only important in tool making, but also in electrotechnology, especially in the construction of inverse feed-back amplifiers. If the output voltage of the amplifier is greater than 10 volts it can be applied directly to the set of plates for horizontal deflection of the cathode ray tube, while the input signal causes the vertical deflection of the cathode ray *via* the amplifier of the oscillograph. If output voltage and input voltage are in the same or opposite phase, a straight line results; in other cases an ellipse from which the relative phase shift can be deduced [6]).

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[6]) On the subject of this and other methods of phase measurements with the help of the cathode ray tube see Philips techn. Rev. **5** 210, 1940.

The measurement of varying wind velocities

In aerodynamics, wind velocities or flow velocities in turbulent air currents are measured from the cooling experienced by thin heated filaments in the air. The temperature variations of the filament cause resistance variations which can easily be converted into voltage variations. The latter can be observed with an oscillograph.

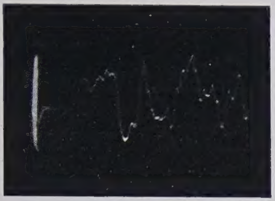

Fig. 12. Variation of the resistance of an anemometer filament in the air current of a fan.

An example of the diagrams thus obtained is given by the oscillogram of *fig. 12* which was recorded in the air current of a fan. Since this current does not have a periodic character, the non-periodic linear horizontal deflection is used. The starting point of the horizontal motion is marked as a white line at the left.

In the following number of this periodical we hope to describe additional examples of practical applications of the cathode ray oscillograph GM 3 156.

THE PERCEPTION OF PITCH

by J. F. SCHOUTEN.

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Although the human ear is capable of breaking up a complex sound into harmonics, which is understandable physiologically by the so-called localization theory, upon superficial listening the ear perceives only a single pitch which is in general that of the fundamental tone. The fact that this is often true even when the fundamental tone is almost or entirely missing has been explained as due to non-linear distortion in the ear. Experiments done in this laboratory have shown that this explanation cannot be correct, and have led to the hypothesis that the low pitch observed is to be ascribed to a collective perception of the higher harmonics. This component of the sound is called the "residue". The low pitch of the residue is found to be correlated with the periodicity of the vibration which occurs due to the conjunction of the higher harmonics. It is physiologically understandable that such a conjunction should occur, considering the limited resolving power of the ear. This is made clear by means of a model. In conclusion several phenomena are discussed, some of which have long been familiar, which can be given a simple explanation by means of the hypothesis of the residue; the sound of church bells is discussed in particular.

Subjective sound analysis

Every periodic air vibration can be decomposed into the sum of a number of sinusoidal vibrations (harmonics) whose frequencies are integral multiples of a fundamental frequency (Fourier series). If only the fundamental frequency is present,and if this is in the audible region, the ear observes the air vibrations as a pure tone whose pitch is determined by the frequency. If in addition to the fundamental frequency the vibration contains a number of harmonics, other perceptions are also experienced. In general one observes a sound to which one ascribes a single definite pitch, and which is distuinguished from a pure tone with the same pitch by a certain timbre. After some practice, however, it is often possible to observe the different harmonics in the sound separately as pure tones, by concentrating the attention sharply upon the different components to be heard. The fact that the ear is capable of such a Fourier analysis was stated a century ago by Ohm (his so-called acoustic law).

The analytic power of the ear can easily be tested with the help of a piano. For instance we strike the note C (fundamental frequency 131 c/s.) In addition to the fundamental tone a series of harmonics is then heard whose frequencies (at least approximately) correspond to the fundamental frequencies of other notes on the piano. These notes are given for the first ten harmonics of C in *fig. 1.* The fact that these harmonics are actually produced by the wire C, can be proved objectively by first soundlessly depressing a key corresponding to one of the harmonics, for instance G′ (third harmonic of C) and then energetically striking the key C. Upon releasing the key C the G′ wire is clearly heard to sound, having been brought into resonance by the third harmonic of C.

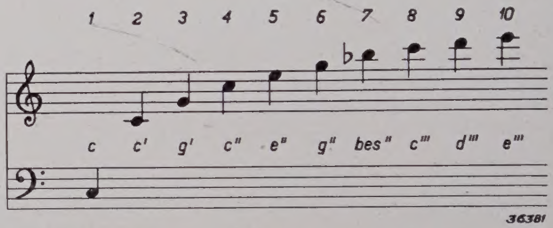

Fig. 1. The harmonics of a note on the piano correspond very closely to the fundamental tones of other notes. They are given here for the first ten harmonics of C.

In order now to observe the third harmonic in the note C subjectively also, the key G′ must first be struck softly (in order to concentrate the attention on that tone) and then released, then the key C must be struck. This can also be done for several higher harmonics up to about the eighth. When one has succeeded in observing the harmonics they are often so suprisingly clear that one is inclined to believe a resonating wire responsible for the sound.

The fact that it is not a question of a resonating wire can be beautifully demonstrated in the following way. The fifth harmonic of C is of course the major third of the C″, this interval is distinguished by the frequency ratio $5:4 = 1.250$. In the equally tempered scale of the piano, however, the major third lies somewhat higher ($2^{4/12} = 1.260$), so that the fifth harmonic of C must be heard slightly lower than the E″ (third of C″). This is actually found to be true. When C and E″ are struck at the same time, upon dying out of the sound the E″ is clearly heard to give a beat with a frequency of 5 c/s, corresponding to the difference in frequency between the two tones.

These experiments illustrate how the harmonics of the vibration of a string are perceived as separate pure tones, each with the pitch corresponding to its frequency. As to the mechanism by which the ear is capable of carrying out such a Fourier analysis, the following hypothesis has been developed. In the cochlea (the spiral cavity of the internal ear) there is a membrane, the basilar membrane; see the diagram in *fig. 2.* Due to a sound vibration of a given frequency a local sensation is felt in the cochlea, in the sense that only a certain more or less limited region of the basilar membrane is brought into motion and only the fibres of the auditory nerve originating in this region are stimulated. The old and much contested localization theory has been beautifully confirmed by experiments on animals in which the electrical voltages were investigated which occur in the cochlea under the influence of sound, and which partially leak off to the outside. It was found that especially for high tones a maximum of the voltage could be observed at different points on the cochlea, depending upon the frequency.

If it is assumed, as is often tacitly done, that each of the nerve fibres will, upon stimulation, cause the perception of a definite pitch, a physiological explanation of subjective sound analysis according to Ohm's law is obtained: a composite sound brings into vibration narrow regions of the basilar membrane determined by the Fourier spectrum of the sound, and each of these regions gives the sensation of a separate tone with a pitch which is determined by the region involved.

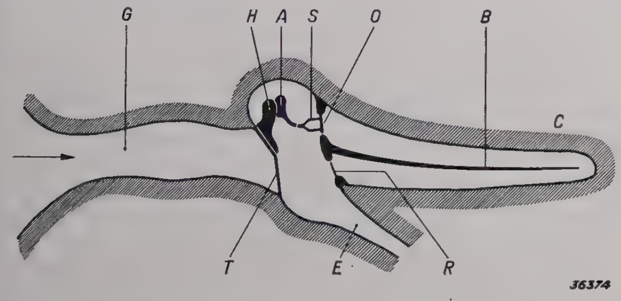

Fig. 2. Diagram of the inner ear. *G* meatus, *T* ear drum, *C* cochlea. The last organ is divided by the basilar membrane *B* into an upper and a lower half. Actually the cochlea, which is drawn extended, is rolled up in a spiral. The bones of the ear (hammer *H*, anvil *A*, stirrup *S*) act upon the oval window *O* in the upper part of the cochlea. The lower part has a circular window *R*, which opens into the Eustachian tube *E*.

We have thus briefly outlined the current conception of subjective sound analysis. Investigations which we are about to describe have led us to the conclusion that these conceptions must undergo a fundamental modification. With the help of an improved formulation of subjective sound analysis, moreover, a number of long familiar paradoxical phenomena in the perception of pitch are provided with a ready explanation. We shall deal with these in the conclusion.

The problem of the missing fundamental tone

We have already stated that in the observation of a sound consisting of different harmonics the ear generally makes no use of its analytic capacity, but only perceives the sensation of a sound with a definite pitch and timbre. This is also the case, for example, when a note on the piano is struck without attention being concentrated especially on one of the harmonics. The pitch ascribed to the sound then simply corresponds to the fundamental tone of the note, which is understandable considering the fact that the fundamental tone is usually much stronger than the accompanying harmonics in the case of the piano.

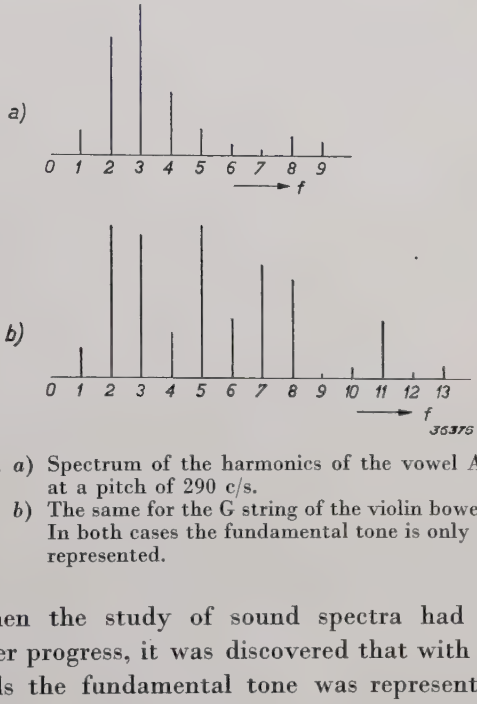

Fig. 3. *a*) Spectrum of the harmonics of the vowel A, sung at a pitch of 290 c/s.
b) The same for the G string of the violin bowed in G. In both cases the fundamental tone is only weakly represented.

When the study of sound spectra had made further progress, it was discovered that with many sounds the fundamental tone was represented to only a very slight degree. This is true for instance for the vowel A and for the G string of the violin bowed in G 1), see *fig. 3.* Nevertheless, we ascribe to these sounds the pitch of the fundamental tone, and we are not in the least inclined to estimate the pitch an octave or a twelfth higher (*i.e.* that of the strongest component of the spectrum).

This remarkable behaviour of the ear was manifested in a still more striking way in telephony. Here it was found that without changing the pitch of the voice of the speaker the whole frequency region below 300 c/s can be cut off. It is just in this region, however, that the fundamental tones

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1) See also: Philips techn. Rev. **4**, 290, 1939 fig. 8 and 9.

of the male speaking voice lie and to some degree also those of the female voice, see *fig. 4*.

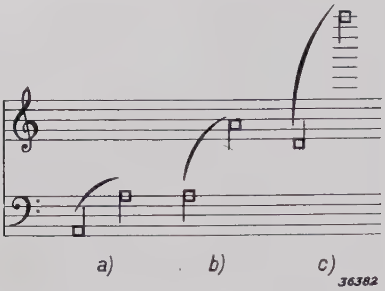

Fig. 4. Region of fundamental tones *a*) of the male and *b*) of the female speaking voice. In telephony (frequency region *c*) the fundamental tones of the man are never transmitted and those of the woman only for high voices.

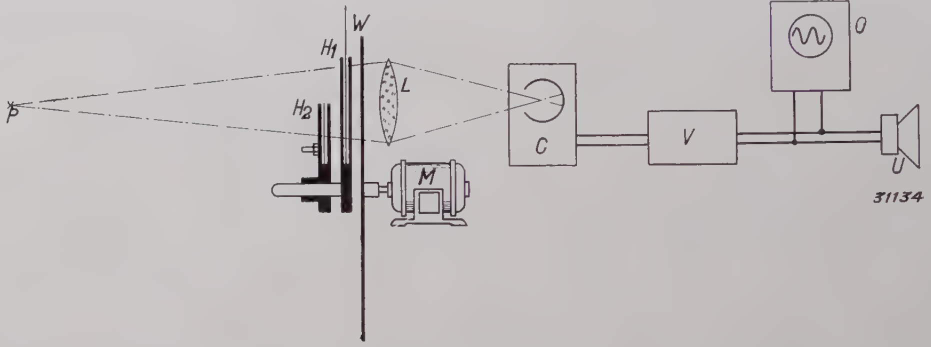

Fig. 5. Optical syren for the production of "synthetic sound". A vibration form which is to be converted into sound is cut out as a paper stencil (such stencils may be seen in fig. 6 to the left), placed in a holder *H*, and homogeneously illuminated by a point source of light *P* placed at a great distance. Behind the holder there is a disc *W* with a number of narrow radial slits, which is driven at an adjustable speed by a motor *M*. The light transmitted through the slits is concentrated by a lens *L* on a photocell *C*. The photocurrents are converted into sound *via* an amplifier *V* and a loud speaker *U*. Since upon rotation of the disc *W* the slits repeatedly scan the vibration form cut out in polar coordinates, the sound obtained also has this vibration form, which may be checked with the cathode ray oscillograph *O*. In front of the stencil holder H_1 is a second holder H_2 which can be rotated, and in which a second stencil can be inserted.

The explanation of the fact that in these cases, in spite of the absence of the fundamental, the pitch of the fundamental is ascribed to the sound has been explained as due to the non-linear distortion which the sound vibrations undergo in the transmission within the human ear [2]). Due to this distortion differential tones, among others, of the components present in the sound will occur. Each pair of adjacent harmonics will give a differential tone with a frequency just equal to that of the fundamental tone (see for instance the spectra in fig. 3). In this way therefore it would be possible that a sound which is lacking objectively (*i.e.* outside the ear) in the fundamental tone should possess this tone in the internal ear. If this phenomenon were really very pronounced, it would even make no difference whether or not the sound contains the fundamental tone objectively; in any case the subjective fundamental tone would always be dominant due to non-linear deformation.

In opposition to this statement it may be remarked that with non-linear distortion it is a case of a phenomenon which must of necessity disappear at sufficiently low sound level, while the phenomenon to be explained — the perception of the missing fundamental — continues to take place at a low level. The statement referred to is, however, entirely refuted by experiments with the optical syren, an instrument which has already been described in this periodical [3]). With this instrument, a diagram of which is given in *fig. 5* with a short description in the text under the figure, it is possible to convert into sound a series of arbitrary vibration forms cut out in the form of stencils, separately or in combination; in particular it is possible to make one of the components disappear while listening to the "synthetic sound" thus obtained, by covering the stencil in question, or to change its phase by rotating the stencil.

In this way a periodic impulse was investigated whose oscillogram and spectrum are given in *fig. 6a*. The fundamental frequency was 200 c/s. This vibration was observed as a sharp sound wih the pitch 200 c/s. By adding to the sound with the help of a second stencil a vibration with a fundamental frequency in a phase opposite to that of the fundamental vibration of the periodic impulse and with a suitable amplitude, the fundamental vibration could be removed from the objective sound (present outside the ear), see fig. 6c. The observed pitch of the sharp sound remained the same. If this is to be explained on the assumption of non-linear distortion in the inner ear, it would still now be possible to compensate the fundamental tone subjectively, by adding the fundamental vibration in suitable phase and amplitude, so that the pitch would have to change. This expectation was not fulfilled, the pitch remained the same, which is all the more remarkable since a subjective compensation is indeed found possible in the investigation of the second and higher harmonics. The separate perception of these harmonics — which is particularly easily realized in these experiments by alternately covering and uncovering the corresponding stencils — can be made to disappear entirely (see fig. 6*c*). At not too high a sound level the subjective compensation of these higher harmonics is found to occur simultaneously with the objective compensation, so that in this case the non-linear distortion can play no appreciable part.

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[2]) Non-linear distortion in the ear is discussed in J. F. Schouten, Synthetic sound, Philips techn. Rev. 4, 167, 1939; see especially p. 172.

[3]) See the article referred to in footnote [2]).

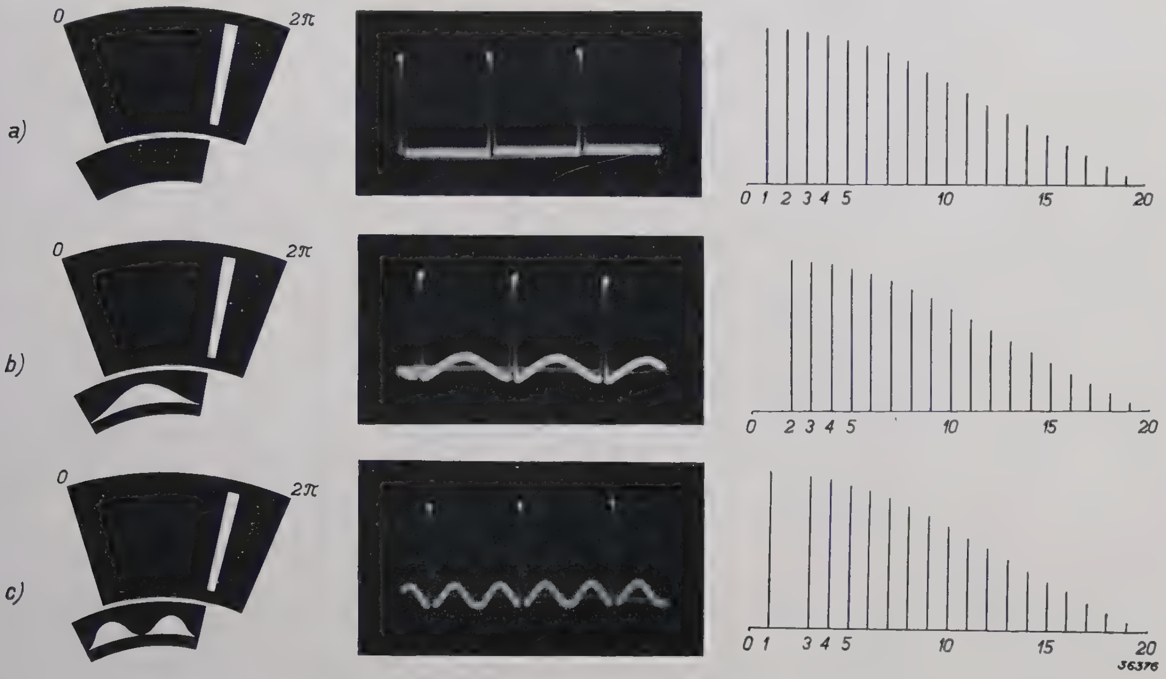

Fig. 6. Stencil, vibration form (recorded with the oscillograph) and spectrum of
a) a periodic impulse whose peaks have a width of 1/20 of the length of a period;
b) the same periodic impulse with (objectively) compensated fundamental frequency;
c) the same periodic impulse with compensated second harmonic.

The hypothesis of the residue

Experiments with the periodic impulse mentioned at a sound level at which, according to the above, it is certain that no non-linear distortion occurs, brought to light the following remarkable behaviour. Whether or not the fundamental vibration was present, a strong sharp sound with the pitch of 200 c/s continued to be observed. Addition and subtraction of the fundamental vibration is heard, however, independent of the sharp sound, as the occurrence and disappearance of a weak, pure tone with the pitch 200 c/s. This is the separately observed fundamental tone; it exactly disappears when the fundamental vibration is compensated objectively. The loudness with which we now hear the fundamental tone in the total sound is also practically the same as when we listen to the fundamental tone alone by covering the stencil of the impulse. Furthermore, upon the addition of an extra frequency of for instance 204 c/s, beats with the fundamental tone are heard which disappear completely when the latter is compensated. The sharp sound of the same pitch, which one continues to hear, on the other hand, exhibits no beats with the added frequency of 204 c/s.

The total sound thus contains two components of the same pitch. That this phenomenon has until now escaped the attention of observers must be ascribed to the difficulty of observing the fundamental tone as a separate weak component in the sound without the above-described means.

As concerns the strong, sharp sound with the pitch of 200 c/s, this is found also to persist, not only when the fundamental vibration is removed from the sound, but also when a number of low harmonics are removed as well. It is, however, very much diminished when the highest harmonics are taken away. We are therefore compelled to assume that a collective observation of these high harmonics is the source of this sharp sound. Such a sound component will be called a residue.

A very simple solution of the problems of subjective sound analysis is furnished by the introduction of the conception of the residue. For complex sounds the rule, assumed in the foregoing to be obvious, is valid, that the pitch ascribed to the sound is that of the loudest component [4]. For all sounds which contain a large number of hamonics the residue will be the component which determines the pitch [5]; the fundamental tone itself then plays an entirely minor part. If it is practically missing, as in the case of the vowel A or the G of the violin, or if it is removed, as in telephony or in the above-described experiments, the residue continues to exist unchanged, so that the pitch remains the same.

In the case of such sounds, and there are after all many of them, it is no longer reasonable to say that the ear carries out a Fourier analysis of the sound. This is, however, true for the lower harmonics, but the most obvious component, the residue, owes its existence to the fact that the group of highest harmonics is not analysed into different pure tones but is observed as a single sound.

Correlation of residue and periodicity

It is not strange that the highest harmonics cannot be observed individually, since they lie very close together; the resolving power of the ear is clearly inadequate to distinguish them from each other. The question arises, however, as to why they form collectively a component of such a low pitch. There are two quantities which may play a part in the explanation of this: in the first place the distance between the harmonics which is equal to the fundamental frequency, and in the second place the periodicity of the collective vibration form of these harmonics. In the case of practically all vibration forms this periodicity (*i.e.* the frequency corresponding to it) is again equal to that of the fundamental vibration. Thus in the periodic impulse used by us (fig. 6*a*) not only the distance between the harmonics, but also the periodicity of several groups of adjacent harmonics is 200 c/s. Vibration forms can also be realised in which the two quantities mentioned differ. The vibration given as an example in *fig. 7a* has the same spacing of the harmonics as the periodic impulse in fig. 7*b*, the periodicity, however, not only of the total vibration but also of each group of adjacent harmonics, is twice as great in the second case as in the first. In observing these vibrations with the help of the optical syren it was found that the residue observed in the second case was an octave higher than the residue in the first case. The pitch of the residue is thus not correlated with the distance between the harmonics but with the periodicity of the collective vibration form of the constituent harmonics.

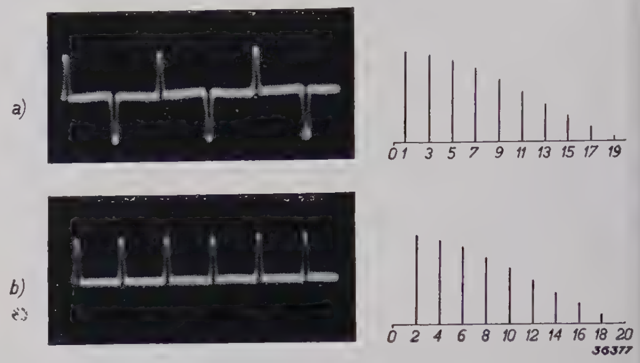

Fig. 7. Oscillogram and spectrum of two vibrations which have the same distance between the harmonics but different periodicity.

How can the existence of the residue be explained physiologically?

For the theory of the mechanism of hearing the most important consequence of the existence of this residue is that motion of a single region of the

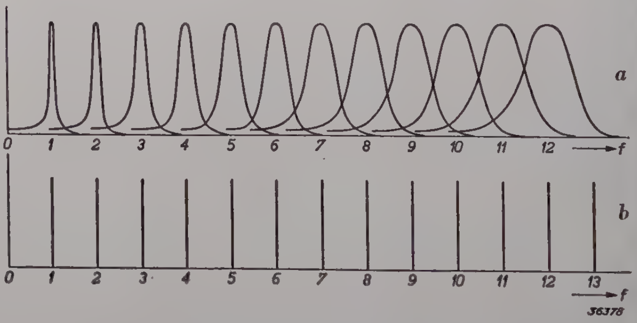

Fig. 8. *a*) Resonance curves of an series of resonators of the type of the basilar membrane. The resonators may be imagined to succeed one another in infinite density.
b) Spectrum of the harmonics of an ideal (infinitesimally narrow) periodically repeated impulse.

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[4] The pitch may also be determined by that component which draws the most attention by contrast with previous sounds or in some other way.

[5] In principle it is possible for more than one residue to occur in a single sound.

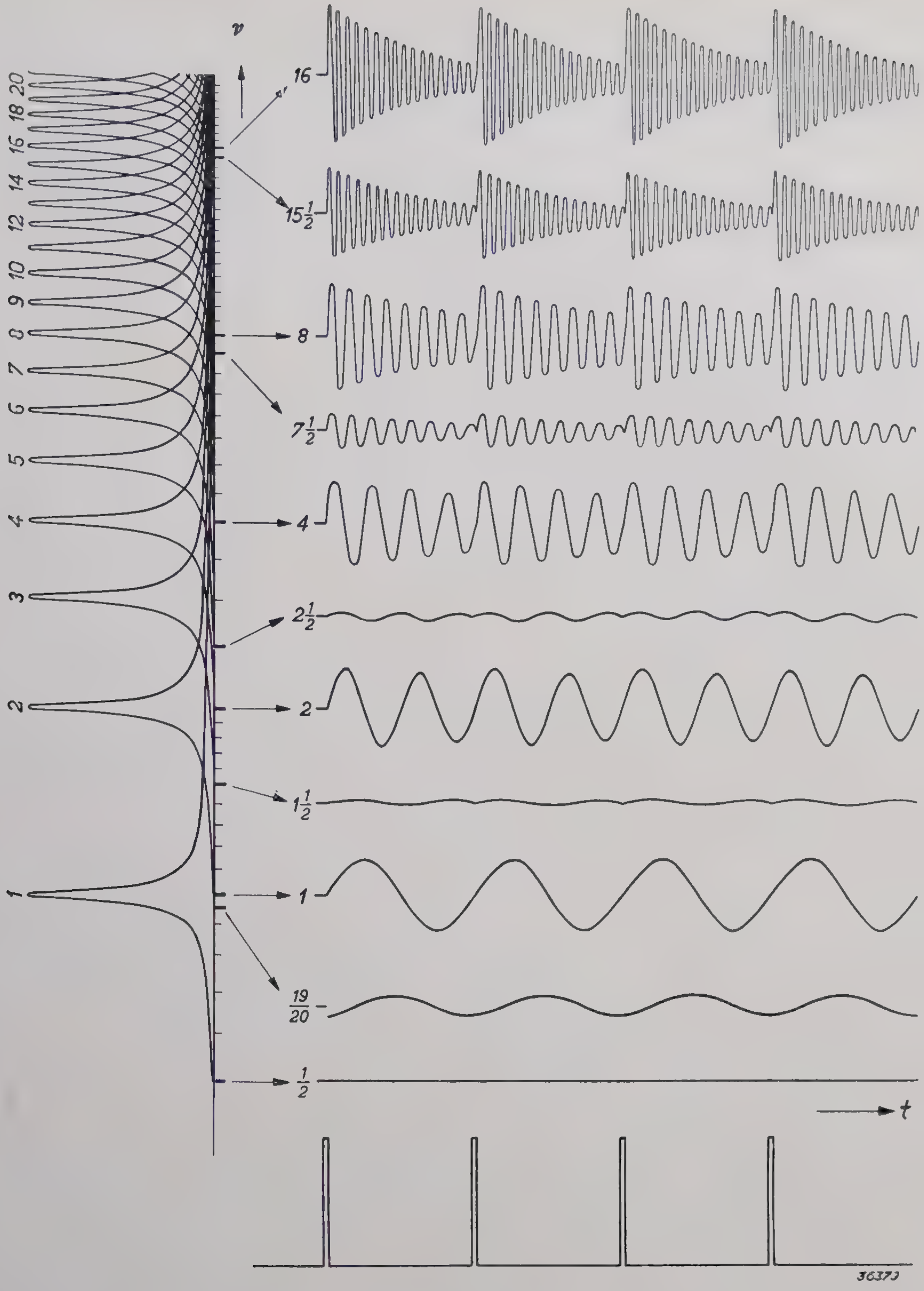

Fig. 9. To the left on a logarithmic frequency scale the excitation curves which indicate for each harmonic of the force (periodic impulse) acting on the model which resonators (characteristic frequency ν) are set in motion and to what extent. The motion occurring is here drawn for a number of resonators (relative characteristic frequency $^1/_2$, 19/20, 1, $1^1/_2$, 2, $2^1/_2$, 4, $7^1/_2$, 8, $15^1/_2$ and 16). It may be seen that in the region of the higher harmonics, where the excitation curves overlap each other very much, the vibration of all the resonators clearly shows the periodicity of the fundamental frequency.

basilar membrane may lead to perceptions of very divergent pitches. The frequencies 2 000, 2 200, 2 400 c/s, etc. will separately cause perceptions of pure tones with pitches of 2 000, 2 200, 2 400 c/s. Together, however, they will lead to the perception of a sharp sound with a pitch of 200 c/s. This means that for determining the pitch it is not the spot on the basilar membrane which moves (nor the particular nerve fibre stimulated) which is the deciding factor. We must rather ascribe to the nerve fibres the power of transmitting not only the quantity of the stimulus (loudness) but also the quality (pitch).

A comparison with the eye shows that this is very well possible from a neurological point of view. The basilar membrane must be considered as an analogue of the retina. The acoustic system in the inner ear represents in the optical analogy a spectroscope which "focusses" vibrations of different frequencies upon different parts of the retina. In the older conception the ear was looked upon as a colour blind eye gazing into the spectroscope: it was able to determine the frequency from the position of the lines, for instance with the help of a scale division. According to the conception made necessary by the theory of the residue, we must, however, consider the ear as comparable to a colour distinguishing eye looking into the same spectroscope. This eye will be able to draw a conclusion about the frequency of the light from the colour, aside from the position of the lines. In a grating spectroscope, in which spectra of different orders may overlap, this eye will even be able to recognize for instance a red line at a position where only blue lines are usually visible. In acoustic language, a component of low pitch (the residue) may originate in a region of the basilar membrane which in the case of pure tones gives the perception of high pitch.

We shall not enter more deeply into the forms in which such a "sense of colour" of the ear might be supposed to be realized ⁶). We shall, however, show that the current conceptions about a local stimulation of the basilar membrane in principle do not exclude the possibility that a periodicity of the sum of a number of harmonics should occur as an actual quantity in this stimulus.

For this purpose we consider a mechanical model of the basilar membrane, which consists of a system of mechanical resonators of progressive characteristic frequency (a kind of harp). For the sake of simplicity we assume that the sensitivity of all the resonators is the same, and that the width of the resonance curves for all resonators is equal to the same percentage of the characteristic frequency, see *fig. 8a*. An ideal periodic impulse is allowed to act upon this system, the spectrum of which is given in fig. 8*b*. Each of the harmonics will set in motion a whole region of resonators to the degree given by the excitation curves which can easily be deduced from figs. 8*a* and *b*. In *fig. 9* on the left these excitation curves are drawn on a logarithmic frequency scale for the first 20 harmonics. For higher harmonics the excitation curves begin to overlap one another more and more, or in other words, one resonator will react simultaneously to a continually larger number of adjacent harmonics. Fig. 9 shows that for low frequencies, where the excitation curves do not yet overlap appreciably, regions of stimulation alternate with regions in which the stimulation is practically zero. The excited resonators here vibrate almost sinusoidally. At high frequencies, however, the seperation between the regions is lost, and the resonators now exhibit vibration forms in which the fundamental period of the periodic impulse is clearly manifested, and which possess exactly the perodicity of the fundamental tone.

If the transmitting mechanism (the nerves) were able to distinguish this periodicity, the existence of the residue as well as its pitch would be explained. In *fig. 10* we give in conclusion a comparison of the objective sound spectrum, the excitation of the system of resonators (the basilar membrane) and the components of the subjective analysis for a periodic impulse of a given width.

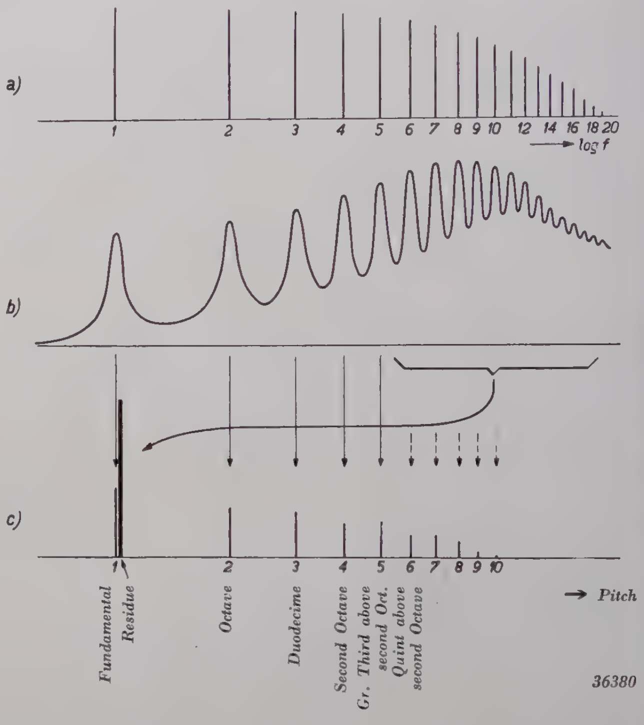

Fig. 10. Diagram of the occurrence of the subjective sound spectrum.
a) The objective spectrum of a periodic impulse with a width equal to 1/20 of the length of the period.
b) The stimulus caused by this impulse on the basilar membrane.
c) The subjective spectrum in which the ear analyses the stimulus represented by (*b*). The stimuli coming from those parts of the basilar membrane upon which the higher harmonics are localized are perceived collectively as a component of low pitch, the residue.

Applications of the hypothesis of the residue

We shall now discuss several remarkable phenomena in which the hypothesis or the residue can be applied.

Anharmonic sounds

While in the case of stringed and wind instruments, in which strings and air columns,

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⁶) See in this connection: J. F. Schouten, Proc. Ned. Akad. Wet. Amsterdam **41**, 1086, 1938; **43**, 356, 1940 and **43** October 1940 (not yet published).

respectively, serve as oscillators, the frequencies of the constituent tones (harmonics) of a sound are integral multiples of the fundamental frequency, this is not the case with musical instruments with less simple oscillators. Examples of the latter are the triangle 7) and, perhaps the most familiar, church bells. The sounds of these instruments whose vibration form is no longer purely periodic, are called anharmonic.

In the case of church bells the remarkable and long familiar phenomenon occurs that the tone by which a bell is known, the so-called strike note does not in general appear in the spectrum of the sound of the bell. While it is possible, by placing tuning forks of the corresponding frequencies upon the bell in a suitable manner, to bring them into resonance in each of the bell's characteristic frequencies, this cannot be done for the strike note. In *fig. 11a* an example of a bell spectrum is given which is characteristic of the bells cast by the Hemony brothers (seventeenth century) 8).

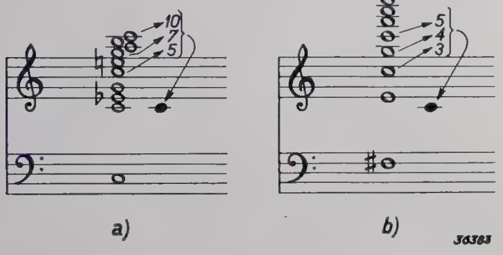

Fig. 11. *a*) Components of the sound of a church bell. The strike note is not to be found among the characteristic frequencies of the bell. Its pitch, however, usually lies very close to that of the second characteristic tone. The fifth, seventh and tenth characteristic tones form a more or less harmonic series, their frequencies are almost exactly in the ratio of 2 : 3 : 4. The strike note may be considered as the residue of these, and it therefore lies an octave below the fifth characteristic tone.

b) Components of the sound of a steel spring (of a kind of Westminster chime). The strike note is here the residue of the third, fourth and fifth characteristic tones, whose frequencies are approximately as 2 : 3 : 4. The strike note thus lies an octave below the third characteristic tone.

To what must the existence of the strike note be ascribed? Its pitch is often almost equal to that of the second characteristic frequency. The latter, however, can clearly be distinguished from the former not only by its quality — the strike note is sharp, metallic, the characteristic notes soft and pure — but also by the fact that the strike note (like the high characteristic notes) dies out relatively quickly and the second characteristic note (also called resultant tone or "hum note") only slowly 9). Another empirical rule has been set up by Rayleigh; this may be formulated as follows. The strike note lies an octave below the pitch of the fifth characteristic frequency. The rule has also been proposed that the pitch of the strike note corresponds to the frequency difference between the fifth and seventh characteristic frequency, and this has been considered from the physical and technical point of view as the reason for explaining the strike note simply as a differential note appearing due to non-linear distortion. The Netherlander Arts 10), however, has shown anew from extensive factual material that Rayleigh's rule is almost always better obeyed than the differential note rule. Moreover, in the case of the differential note hypothesis, the question remains unsolved as to why just the fifth and seventh characteristic notes should give a strong differential note, and why the strike note has a different sound than the other components.

The hypothesis of the residue throws some light on this problem. It may be expected that when a number of frequencies in an anharmonic spectrum chance to be integral multiples of a certain "fundamental frequency", the latter will become audible as the residue. In the case of the bell, see fig. 11*a*, the frequency ratio between the 5th, 7th and 11th characteristic frequency is in good approximation 2 : 3 : 4. These characteristic frequencies therefore would actually result in a residue whose pitch would lie an octave below the 5th characteristic tone. It is now possible to understand also why in some "ugly" bells the strike note is less clear or almost lacking: this must be the case when there is no series of characteristic frequencies in the bell spectrum which can be expressed reasonably approximately by whole numbers.

A nice confirmation of this conception was found in an investigation of the sound of a series of steel springs (a kind of Westminster chime). In fig. 11*b* a spectrum of this is reproduced. The strike note was, within 1 per cent, an octave below the third

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7) See fig. 11 of the article referred to in footnote 1).

8) This bell spectrum has the same composition for bells of different pitch, as far as the intervals of the components are concerned. In playing a melody on a chime of bells the same chord (in this case mainly the minor chord of the strike note) is heard in every sound. Debussy made an elaborate use of this property in imitating the sound of bells in his piano prelude "La cathédrale engloutie".

9) Here we find the perception, clearly pronounced, of two simultaneously occurring components of the same pitch and different quality, just as was found for the fundamental tone and the residue of the periodic impulse. This was already referred to by P. J. Blessing, Phys. Z. **12**, 597, 1911.

10) J. Arts, J. Acoust. Soc. Amer. **9**, 344, 1938 and **10**, 327, 1939.

characteristic note. The third, fourth and fifth characteristic frequencies in this case had approximately the ratio: 2 : 3 : 4.

Sounds made anharmonic artificially

It is possible to shift all the frequencies of a sound by the same number of c/s. If the sound was originally harmonic, *i.e.* if the frequencies of the components were integral multiples of the fundamental frequency, it becomes anharmonic by this treatment. The experiment can easily be carried out with the help of an apparatus for carrier-wave telephony [11]). A low frequency signal of frequency f is modulated on a carrier wave of frequency ν. The frequency $\nu + f$ obtained by filtering is modulated with a carrier wave of frequency $\nu - \Delta$ and a low frequency signal is again obtained which now has the frequency $f + \Delta$. If the repeatedly used periodic impulse with the fundamental frequency of 200 c/s is treated in this way with Δ made equal to 40 c/s, the components become 240, 440, 640 ...... c/s.

What pitch perception must now be expected of this sound? If the pitch were determined by the fundamental tone itself, it would then have to be $\Delta = 40$ c/s, *i.e.* a minor third, higher. If the pitch were determined by the differential tone, it would have to remain constant, since the distance between the harmonics is not changed by the shift. Neither of these two cases actually occurs; the pitch does change, but very little, less than half a tone. In the case of music also we found the same slight increase in pitch. This increase was measured by the comparison of two identical gramophone records, one of which was played at the normal speed and the frequencies shifted by a small amount Δ, while the other was played without the frequency shift but at such a speed (*i.e.* with multiplication of all the frequencies by such a factor), that the sound of the two records corresponded as well as possible in pitch. The result can be described by saying that the pitch is shifted relatively to the same extent as would be expected in the case of pure tones with frequencies of 1 000 to 2 000 c/s.

With the help of the residue this behaviour can be understood. Assume that a residue of the pitch 200 c/s is mainly caused by the 9th, 10th and 11th harmonics with the frequencies 1 800, 2 000, 2 200 c/s. If these frequencies are shifted to 1 840, 2 040, 2 240 c/s, respectively, they may be considered to be approximately the 9th, 10th and 11th harmonics of the frequency 204 c/s. The residue is thus shifted from 200 to 204 c/s, *i.e.* proportionally by the same amount as the frequency 2 000 c/s is shifted when $\Delta = 40$ c/s.

Partial deafness

In conclusion we should like to point out a remarkable consequence of the new hypothesis. In the case of certain defects in the sense of hearing it is only the observation of certain frequency regions which is defective. A person who is "partially deaf" in this way for all low frequencies, below 1 000 c/s for example, will not be able to hear the harmonics lying in this region, but he will be able to hear a residue with a pitch of say 200 c/s. Still more paradoxical is the behaviour which must be expected in the case of deafness for high tones. Since the high tones in a complex sound cause the residue, deafness for high tones will result in the fact that upon listening to sharp sounds one of the lowest components of the sound, the residue, will not be heard. These phenomena have not yet been investigated experimentally.

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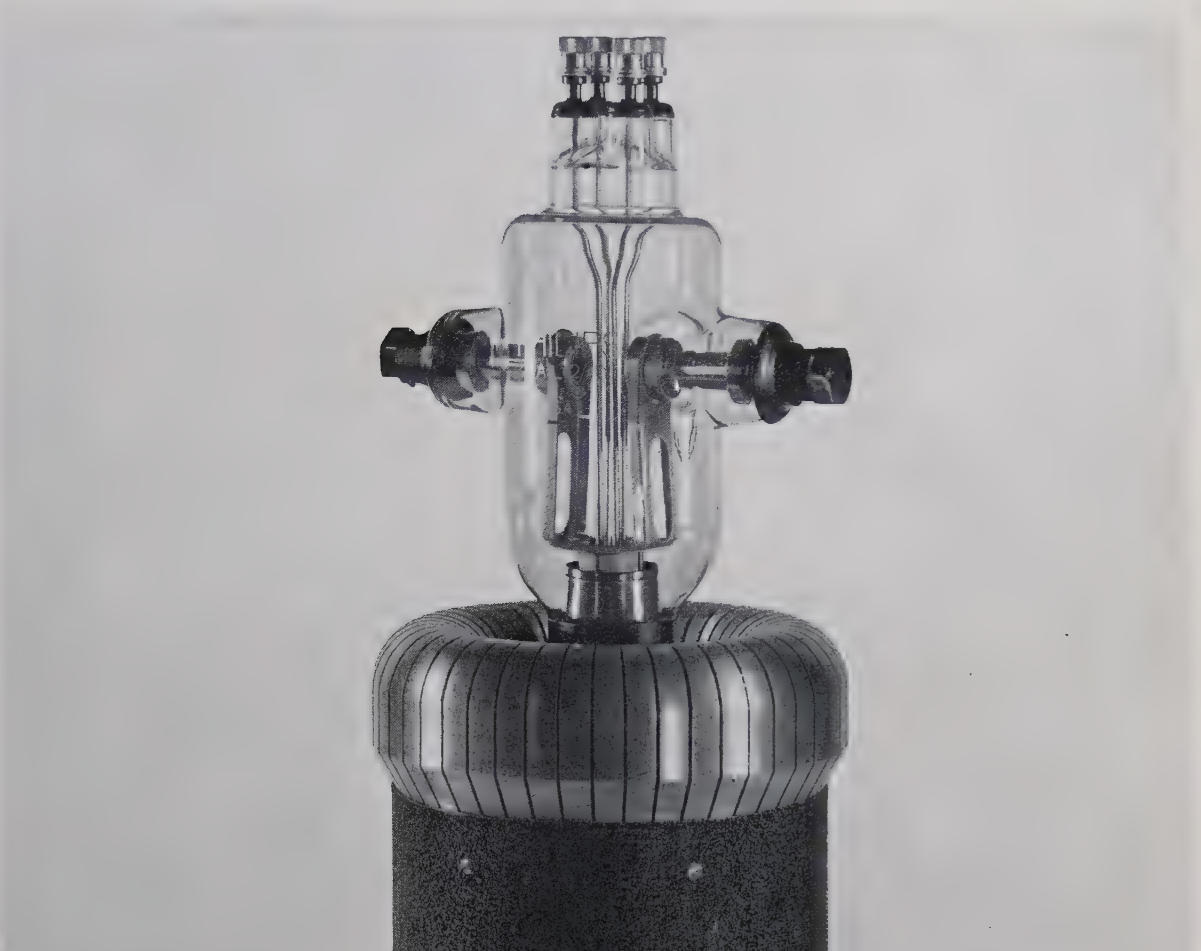
[11]) H. Fletcher, Phys. Rev. **23**, 427, 1923. In this article it is described how the complex sounds of musical instruments lose all musicality with values of Δ which are not too small.

THE FORCED AIRCOOLED TRANSMITTING VALVE TAL 12/10



The maximum allowable anodedissipation of 4 kW is conducted from the anode by about 7.5 m^3 air per minute. In class C telegraphy adjustment the output power amounts to 10 kW and to about 6 kW in the carrier wave for anodemodulation. The air is circulated by means of a blower, enters through a number of slits. Overall length 446 mm, diameter at the cooling slits 195 mm.

NATURAL ILLUMINATION AND VISIBILITY AT NIGHT

by P. J. BOUMA.

535.241.44: 612.843.612: 628.92.024

In connection with air-raid defence it is important to know what illumination is present and what the possibility of observation is at night due to natural sources of light (moon, planets, stars, etc.). Some data are given about these facts, and these data are compared with corresponding data which were used in prescribing the legally permissible black-out illumination.

The latest branch of illuminating technology, that of black-out illumination [1]), (also called air-raid defence illumination) has stimulated fresh interest in the question of the possibilities of observation which exist at night in the complete absence of artificial illumination.

In order to judge this question two types of data are important: data about the light sensitivity of the eye, and data about the intensity of the natural illumination prevailing at night (moon, stars, planets, etc.). In the following we shall give a short survey of these data.

The absolute threshold values of the eye

Under the most favourable conditions it is possible for the eye to observe a point source of white light which produces an illumination intensity of 10^{-9} lux on the eye. If atmospheric conditions were without disturbing influence, this illumination intensity could be obtained by placing a candle at a distance of 32 km (on a tower 80 m high in order to project above the horizon)!

The pupil of the eye (8 mm in diameter) receives under these circumstances about 5×10^{-14} lumen. It is interesting to compare this with the sensitivity of photocells. When photocells are used very special measures must be taken in order to measure light fluxes of the order of 10^{-11} lumen [2]). The great sensitivity of the eye is even more clearly realized when we calculate the minimum amounts of energy necessary to cause an impression of light. If we use monochromatic light of the wave length for which the dark-adapted eye is most sensitive (5 100 Å), an energy of 4.4×10^{-17} watt or 4.4×10^{-10} erg per sec is found to be sufficient to give an impression of light [3]). An erg is approximately the amount of energy which is liberated when a weight of 1 mg is allowed to fall 1 cm. This small amount of energy, if completely converted into light of 5 100 Å, would be capable of producing in the eye a light impression during a period of 72 years. We would then receive more than 100 light quanta per second. The lowest brightness which the eye can observe under the most favourable circumstances amounts to about 3×10^{-10} stilb $= 3 \times 10^{-6}$ c.p./m². Such a brightness would occur if we illuminated a snow surface with a candle placed about 300 m above it; the brightness of the snow would then be a factor 30 lower than it is during a clear moonless but starry night.

Astronomical light units

In order to be able to compare the low values of illumination intensity, brightness, etc. which prevail in black-out problems, with those which occur at night naturally, we must make a few statements about the light units which are used in astronomy. There is danger of confusion in this subject because a quite different nomenclature is used in astronomical literature from that used in illumination technology. We shall therefore confine ourselves to a single universally applied concept, namely that of the magnitudes of the stars, and we shall otherwise use the nomenclature of ordinary light technology.

The magnitude of a star is a logarithmic measure of the illumination intensity which a star produces on the eye, and may be related to the light technological quantities in the following way:

a) a difference of 5 magnitudes means a factor 100 in the illumination intensity on the eye [4]);

b) a star of the first magnitude gives 8.3×10^{-7} lux at the eye.

If we also keep in mind that the astronomer ascribes a greater value to the magnitude, the weaker the star, we can formulate the relation between magnitude m and illumination intensity E in the following way:

$$m = -14.2 - 2.5 \log E$$

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[1]) See also Philips techn. Rev. **4**, 15, 1939, and **5**, 93, 1940.

[2]) Philips techn. Rev. **4**, 66, 1939.

[3]) The sensitivity of the eye expressed in energy is practically the same as that of the ear. The maximum ear sensitivity is 10^{-16} watt/cm² (see the article referred to in footnote [4])), while a value of 0.3 to 0.5 cm² is given as the effective area of the ear.

[4]) The magnitude plays the same rôle in astronomy as the decibel (phon) in acoustics (See Philips techn. Rev. **2**, 47, 1937). A magnitude corresponds to 4 decibels.

The following table gives several examples

Sun	$m = -26.7$	$E = 100\,000$ lux
Full moon	$m = -12.5$	$E = 0.2$ lux (1 candle at 2.2 m)
Venus (maximum)	$m = -4.3$	$E = 1.1 \times 10^{-4}$ lux (1 candle at 100 m)
Sirius	$m = -1.6$	$E = 9 \times 10^{-6}$ lux (1 candle at 330 m)
Star of first magnitude	$m = 1$	$E = 8 \times 10^{-7}$ lux (1 candle at 1100 m)
Star of 6th magnitude	$m = 6$	$E = 8 \times 10^{-9}$ lux (1 candle at 11 km)
Star of 21st magnitude	$m = 21$	$E = 8 \times 10^{-15}$ lux (1 candle at 11 000 km)

The sun therefore gives 10^{13} times as much light to the eye as the weakest star visible with the naked eye (6th magnitude). The most powerful telescopes, aided by photography as integrating factor, add to this range of 33 magnitudes an additional range of 15 magnitudes, or a factor of 10^6 (21st magnitude).

Brightness of the night sky; illumination intensity during a clear moonless night

During a clear moonless night an average illumination intensity of about 3×10^{-4} lux occurs upon a horizontal plane. This value, derived from measurements carried out at different times and at different places on the earth's surface, is still a factor 10 to 20 smaller than the illumination level permitted in the black-out 1). The expressions current in England such as "artificial starlight", "starlight lamps", etc. are thus rather unfortunately chosen.

To what is this illumination intensity due? One would be inclined to say: in the first place to the fixed stars which can be observed all about with the naked eye. Remarkably enough, however, this conclusion is quite incorrect. In the following we shall make an estimation of the different contributions.

a) *The direct light of the fixed stars visible to the naked eye*

It is known from star counts how many stars of each magnitude there are. If one calculates the number of stars of the first magnitude to which each of these groups is equivalent, and adds together in this way the amounts of light of all the stars to the 6th magnitude, one finds that all the visible stars together are equivalent to 148 stars of the first magnitude. If all these stars were at the zenith they would give an illumination intensity of 1.23×10^{-4} lux. This value must, however, be divided by two, since only half the stars are above the horizon, and then again divided by two, since the stars are distributed over the whole celestial hemisphere, so that the contribution of each star must be multiplied by the cosine of its angle of incidence (distance from the zenith). For the direct light of the fixed stars visible to the naked eye we obtain in this way a total contribution of $\mathbf{0.3 \times 10^{-4}}$ **lux.**

b) *The direct light of the fixed stars which are not observed individually by the naked eye*

Telescopic observations show the distribution of the fixed stars among the different magnitudes up to the 21st magnitude. The contribution of still weaker stars may be neglected. All stars from the 6th to the 21st magnitude are found to be equivalent to about 540 stars of the first magnitude. Upon application of the two corrections mentioned, we find for the total contribution to the illumination intensity $\mathbf{1.1 \times 10^{-4}}$ **lux.** The "invisible" stars thus furnish four times as large a contribution as the "visible" ones!

c) *The direct light of the planets*

When the magnitudes of the different planets are examined, the first conclusion will be that their contributions may by no means be neglected compared with that of the fixed stars. The magnitudes of Mercury, Venus, Mars and Jupiter are, respectively,

$$-0.9, \quad -4.30, \quad -1.85, \quad -2.29$$

If they were all together at the zenith the four planets would furnish a contribution of 1.45×10^{-4} lux, thus about as much as all the fixed stars together!

Actually they contribute much less, since at our latitude they are never at the zenith, while, moreover, Mercury and Venus never reach greatest height during the hours of darkness. The maximum contributions of the planets mentioned during the night amount at our latitude to about the following 5):

Mercury	4×10^{-7} lux,	Venus	4.8×10^{-5} lux,
Mars	1.1×10^{-5} lux,	Jupiter	1.6×10^{-5} lux.

Thus a total of 7.5×10^{-5} lux.

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5) Mars and Jupiter at the mean opposition, Venus and Mercury in their most favourable phases, as far as these may occur at least 1 hour after sunset.

Since these maxima, especially in the case of Mercury and Venus, prevail for very short times, the average contribution is much less still; it is estimated to be about **0.1×10^{-4} lux.**

d) The scattered light of stars and planets

The order of magnitude of the part of the starlight which is scattered by the atmosphere can be estimated by noting that in this country (the Netherlands) the brightness of the unclouded sky in the summer at noon is an average of about 0.4 stilb, and would therefore furnish about 12 600 lux on the earth. This is about one eighth of the contribution furnished by the direct sunlight. If the same rule holds for scattered starlight, we obtain a contribution of **0.2×10^{-4} lux.**

e) Other sources

The contributions mentioned until now give a total of 1.7×10^{-4} lux, so that about **1.3×10^{-4} lux** must be ascribed to other sources (zodiacal light, northern lights, unknown sources). The table below gives a survey of the values found.

	E	%
a) Direct light of "visible" fixed stars	$0.3 \cdot 10^{-4}$ lux	10
b) Direct light of "invisible" fixed stars	$0.1 \cdot 10^{-4}$ lux	37
c) Direct light of planets (Mean valve)	$0.1 \cdot 10^{-4}$ lux	3
d) Scattered light of stars and planets	$0.2 \cdot 10^{-4}$ lux	7
e) Other sources	$1.3 \cdot 10^{-4}$ lux	43

About 13 per cent of the light thus comes directly from the visible sources, the other 87 per cent is observed as the brightness of the night sky itself. From this we calculate an average brightness of the night sky of 8.4×10^{-9} stilb, in good agreement with direct measurements of this quantity. This brightness is higher than the lowest observable brightness by a factor of about 30. It also shows why we can only observe stars to the 6th magnitude with the naked eye, and not to the 8th, as would follow from the threshold value of the eye mentioned at the beginning.

Possibilities of observation at these low levels of brightness

In order to obtain an idea of the possibilities of observation at very low levels of brightness, we shall mention several quantities which characterize the power of the eye under three different conditions:

I) with full moon,
II) at the illumination level which is considered just permissible in a black-out,
III) under a clear moonless sky with stars.

The illumination intensities on the ground amount to about the following for these three cases:

I) 0.2 lux, II) 0.004 lux, III) 0.0003 lux.

If it is assumed that the average reflection coefficient of the earth is about 15 per cent, the eye of a pilot will become adapted to the following levels of brightness:

$$\text{I) } 10^{-2} \text{ c.p./m}^2,$$
$$\text{II) } 1.9 \times 10^{-4} \text{ c.p./m}^2, \text{ III) } 1.5 \times 10^{-5} \text{ c.p./m}^2.$$

The sensitivity to contrast of the eye under these conditions can best be characterized by the percentage contrast which large dark objects must have with the surroundings, in order to reach the limit of observability for the pilot. As laboratory values we find:

I) 16 per cent, II) 46 per cent, III) 74 per cent.

Since in practice observation is always considerably less favourable, we may draw the following conclusion: large objects which are considerably darker than their surroundings can never be observed by a pilot by the light of the moonless starry sky; at the level of brightness II of the legally permissible black-out illumination they may under very favourable conditions just reach the limit of observability, while by moonlight, dark objects can often be observed.

The situation is quite different for objects which are considerably brighter than their surroundings. This case will be avoided as far as possible by blacking out; such objects will, however, occur on clear moonless nights. It may be remarked that when the eye is sufficiently adapted, the trees and the roofs of houses may very clearly be seen against the bright sky. The pilot will therefore also be able to observe large objects clearly if their brightness is at all comparable with that of the night sky. The situation may occur in the case of surfaces of water in which the sky is reflected. Experience has indeed shown that even on moonless nights coastal regions, lake regions, broad rivers are clearly observable. Very light coloured roads (with a reflection coefficient of 25 to 30 per cent) may under the most favourable circumstances still just reach the limit of visibility.

Concerning the possibility of recognizing objects it may be remarked that the visual acuity for objects which are a factor two brighter than their surroundings amounts to about the following for the three levels of brightness mentioned:

I) 4′, II) 40′, III) 1° 50′;

for objects which are a factor ten brighter than their surroundings:

I) 2′, II) 20′, III) 55′.

If we assume that in practice these figures lie about a factor two higher than the laboratory values here given, this means that by moonlight and at an altitude of 500 metres the pilot can still observe details of 1.2 and 0.6 metres, respectively. At the level II) of the blacked out city care must be taken that such objects do not occur. On a moonless night only those objects which are very much brighter than their surroundings can be observed. For these objects details of 16 metres will lie on the limit of distinguishability from an altitude of 500 metres. This estimation agrees with the fact that narrow brooks cannot be seen, while broad rivers, on the other hand, can be very well followed along their whole course.

In conclusion it must be noted that the illumination intensity which a point source of light must throw upon the eye of the observer in order to reach the limit of visibility was found in laboratory experiments to be the following:

With absolutely dark blackground 10^{-9} lux.
With adaption level I) 3×10^{-8} lux, II) 6×10^{-9} lux, III) 2×10^{-9} lux.

The practical values lie about a factor 6 higher, thus I) 1.8×10^{-4} lux, II) 3.6×10^{-8} lux, III) 1.2×10^{-8} lux. It is quite clear from these values that, as the general illumination level becomes lower, we must be more and more careful to avoid the occurence of small points of light: on a moonless night it is possible to observe light points 15 times weaker than by full moon!

THE MEASUREMENT OF VERY SMALL PHASE DISPLACEMENTS

by C. G. KOOPS.

621.317.374 : 621.319.4

For various applications in the sphere of telephony, condensers and coils are needed with extremely small losses (phase displacements of the order of 10^{-5} to 10^{-6} radians). In order to measure such small loss angles in one of the ordinary bridge connections, standard condensers are necessary, which themselves have practically no losses. A standard condenser is here described in which the phase displacement has been successfully limited to several times 10^{-8}. The standard condenser has the character of a so-called three-point capacity: the capacity between two of the three electrodes only is free of losses. The two other capacities are rendered harmless for the purposes of the measurement by means of an auxiliary bridge connection and by the application of a substitution method. The influence of the drift of the capacities which have been rendered harmless is limited by means of a second auxiliary bridge connection. Consideration of the possible errors of measurement shows that with the measuring arrangement arrived at in this way phase displacements can be determined with an absolute accuracy of 3×10^{-8}.

In alternating current technology when one speaks of self-inductances and capacitances, impedances are meant in which the current lags or leads 90° in phase with respect to the voltage. In the coils and condensers intended for the practical realization of these impedances the phase shift is always a certain angle δ, depending more or less upon the frequency, smaller than 90°. While in the case of a phase shift of exactly 90° the impedance dissipates no energy, when the phase angle is smaller this is no longer true, and it is for this reason that the phase displacement from 90° is also called the loss angle. Phase displacement is usually indicated by its tangent; for small values of δ, $\tan\delta \approx \delta$ (in radians).

Wherever self-inductances or capacitances are applied technically attempts will be made to keep the losses, and thus the phase displacement δ, low. The requirements thereby made are very different. In the case of power condensers, for example, which are used for the improvement of the power factor, it is already enough if the phase displacements are not greater than several times 10^{-3} or 10^{-2}. Considering the other losses occurring in high power installations, this energy loss is practically unimportant, and the heat development which accompanies it can relatively easily be controlled. In other cases on the other hand, particularly in telephony, in the condensers and coils for filters and in the coils for loaded cables, the problem is to limit the losses to the very minimum. This is particularly important in the case of the filter elements, in order to be able to make the boundary of the transmitting region as steep as possible; in the case of coils for loaded cables, in order to make the damping of the cable small and at the same time the distance to be bridged (without amplification) as large as possible.

The reduction of the phase displacement of these coils and condensers is one of the most important branches of development in the subject of telephony. Because of this fact one must be able to measure very small phase displacements with great accuracy in order to detect causes of losses and to test improved constructions. The installation which has been constructed for the purpose in this laboratory will be described in the following. It is first necessary to discuss briefly the nature of the phase displacement and several concepts used in this connection.

Interpretation of the phase displacement

In order to interpret the phase displacement by means of familiar concepts in alternating current technology an equivalent circuit can be made for the coil or condenser, *i.e.* connections which (at the frequency in question) have the same electrical properties, but consist only of ideal self-inductions, capacities and resistances (in an ideal resistance current and voltage are exactly in phase). Two equiv-

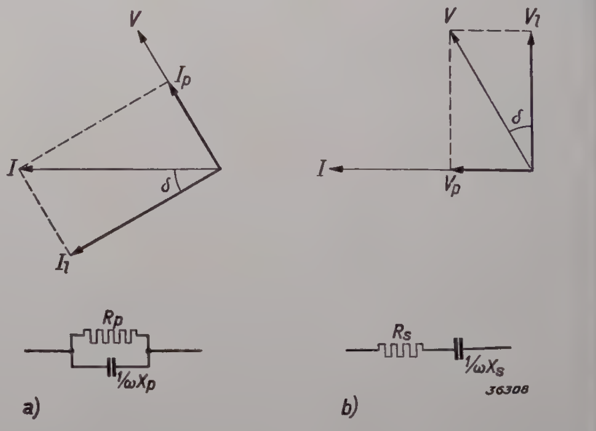

Fig. 1. The vector diagram of a condenser with phase displacement δ can be interpreted in two ways, which lead to two different equivalent circuits. The losses are thereby expressed in a certain parallel conductance $1/R_p(a)$ or a certain series resistance $R_s(b)$.

alent circuits are possible, and these are shown in *fig. 1* for the case of a condenser. In the vector diagram fig. 1*a* the current vector I is divided into two components I_p and I_l, which are parallel and perpendicular, respectively, to the voltage vector V. This vector diagram may be considered as a diagram of a connection in parallel of an ideal resistance $R_p = V/I_p$ and an ideal condenser with the reactance $X_p = V/I_l$. It may be seen that

$$\tan\delta = I_p/I_l = X_p/R_p \quad . \; . \; . \; . \; . \; . \quad (1)$$

$1/R_p$ is called the effective parallel conductance. The smaller it is, the more closely the ideal case ($\delta = 0$) is approached.

In the vector diagram fig. 1*b* the voltage vector V is divided into two components V_p, V_l. This is the diagram of a connection in series of an ideal resistance $R_s = V_p/I$ and an ideal condenser with the reactance $X_s = V_l/I$. Thus

$$\tan\delta = V_p/V_l = R_s/X_s \quad . \; . \; . \; . \; . \quad (2)$$

R_s is called the effective series resistance. For the ideal case it approaches zero.

If in practice one is concerned with a connection in parallel of several condensers, it is often easy to use the method of representation of fig. 1*a*, since then all the parallel conductances $1/R_p$ may simply be added together, like all the "effective" capacities $1/\omega X_p$. In connections in series it is easier for a similar reason to use the interpretation according to fig. 1*b*. The figures and symbols are also valid *mutatis mutandis* for a self-induction.

When δ is very small, *i.e.* when only very small losses occur in the condenser or the coil, $I_l \approx I$ and $V_l \approx V$. Then

$$X_p = \frac{V}{I_l} \approx \frac{V}{I} \approx \frac{V_l}{I} = X_s = X.$$

In that case one may speak of a definite capacity $C = 1/\omega X$ or self-inductance $L = X/\omega$, practically independent of the frequency. The quantities R_p and R_s, however, will in general depend fairly closely upon the frequency.

If a coil and a condenser are connected in series the diagram of fig. 1*b* may again be applied for each, and the series resistance of the resulting equivalent circuit is the sum of the two separate series resistances R_s. This sum can be measured directly when the condenser is variable and has been brought into resonance with the coil; in that case of course current and voltage are in phase. If the value of R_s for the condenser is known, it is found for the coil in this way, and it is clear that in arranging a method of measuring its phase angle one may decide to investigate either coils or condensers. Our method of measurement was developed for condensers, and we shall in the following speak only of condensers.

General method of measurement

For the investigation of condensers bridge connections are universally used which permit very sensitive and accurate measurements. In the course of time a large number of such connections have been developed, each of which has its particular advantages and disadvantages. They have, however, one property in common: they always give the result in the form of the difference in tanδ of the condenser being examined and that of a comparison condenser. We shall explain this for a simple bridge, the equal-armed bridge represented in *fig. 2*.

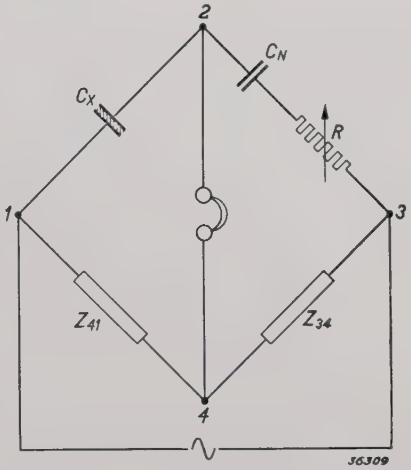

Fig. 2. Equal-armed bridge for the determination of the phase displacement of a condenser C_X. C_N is a standard condenser. The impedances Z_{41} and Z_{34} are made equal, the bridge is then brought into equilibrium by the adjustment of C_N and R.

The condenser to be investigated C_X may have a certain phase displacement δ_X, the variable standard condenser C_N a phase displacement δ_N. R is a variable resistance, Z_{41} and Z_{34} are two impedances which are made accurately equal to each other with regard to magnitude and phase angle. If an A.C. voltage is applied to the points *1* and *3* with the frequency for which δ_X is required, C_N and R_N can be so adjusted that the points *2* and *4* take on the same potential. This is controlled by means of a detection apparatus connected between *2* und *4*, an amplifier with telephone, for instance. With the correct adjustment the impedance between *1* and *2* is equal to that between *2* and *3* with respect to magnitude and phase angle, *i.e.* the effective series resistances of both arms are the same and the effective reactances also; therefore

$$R_X = R + R_N,$$
$$C_X = C_N.$$

From this it follows with equation (2) that

$$\tan \delta_X = R\omega C_N + \tan \delta_N \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From the quantities to be read off R, C_N, ω therefore, it is only possible to calculate the difference $\tan \delta_X - \tan \delta_N$. Similar considerations lead to corresponding results for other bridge connections.

In order to be able to determine $\tan \delta_X$, $\tan \delta_N$ must either be known or else be so small that it may be neglected as compared with inevitable measuring errors. In practice the latter case will always be aimed at, because $\tan \delta_N$ may vary somewhat with the temperature, etc. and with a large value of $\tan \delta_N$ it would be necessary to know these fluctuations accurately and take them into account.

If for instance C_X is a paper condenser in which $\tan \delta$ may amount to several times 10^{-3} or even to 10^{-2}, an air condenser or a good mica condenser may be used as standard C_N. Such condensers have a $\tan \delta_N$ of the order of magnitude of 10^{-5} to 10^{-4}, which is small enough to be neglected, since the phase displacement of paper condensers may vary as much as 5 per cent with the temperature, the humidity of the air, etc., and it is therefore of no use to measure it more accurately.

If, however, one is concerned with smaller phase displacements, the requirements made of the standard condenser are more severe. In our case it was required that it should be possible to determine the phase displacement of a (variable) air condenser with an absolute maximum error of 10^{-6}. This meant, therefore, that we needed a standard condenser (or several standard condensers, since the measurement had to be carried out at different values of the capacity) with a value of $\tan \delta_N$ smaller than 10^{-6}.

Construction of the standard condenser

How can a condenser be constructed with such slight losses? This is in fact again the problem which formed the point of departure of this article, namely the problem of reducing the phase displacement of a condenser to the extreme minimum. The difference is only that in constructing a standard condenser we are not bound to the same degree by consideration of expense and size, etc., as in the construction of condensers and coils for technical use.

The physical causes of losses in condensers are chiefly the following:

1) The ohmic resistance of the connections and condenser plates, which would already involve a dissipation with direct current, while at higher frequencies the dissipation becomes still greater due to the eddy currents which appear.
2) A conductivity of the dielectric (insulation resistance for direct current is not infinitely high).
3) A conductivity in limited regions of the dielectric, for instance in parts with lattice disturbances in crystals, or in semi-conductive impurities in an otherwise insulating dielectric. These losses are not manifested with direct current, but at sufficiently high frequencies they may play a very important part.
4) If the dielectric contains polar molecules, *i.e.* molecules which form permanent dipoles, such, for example, as the molecules of water, these molecules tend to orient themselves in the direction of the applied electric field. With alternating current the dipoles must therefore turn back and forth with the frequency of the voltage. In doing this they experience a kind of frictional resistance in the medium, which results in the dissipation of energy. At very low frequencies (direct current) these losses disappear, and the same is true at very high frequencies, since in that case the dipoles are no longer able to follow the alternations, and thus remain practically at rest.

If it is desired to decrease the phase displacement of a condenser to the utmost, provision must therefore be made for very small resistance of contacts and connections, and a substance must be chosen as dielectric which everywhere possesses a very low conductivity and which contains no polar molecules. These requirements are very well satisfied by dry, dust-free air; the molecules of air are not polar and the insulation resistance is very high when the voltage to be applied remains below the value at which ionization by collision begins to occur.

There are, however, other things to consider. We must now assemble the electrodes of the condenser

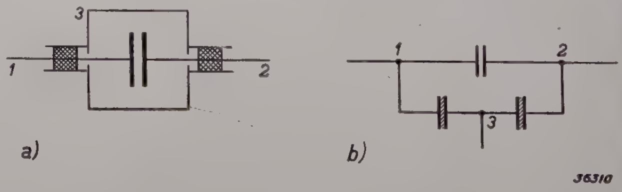

Fig. 3. Principle of the standard condenser. The electrodes *1* and *2* are so assembled (*a*) that no leakage current can flow directly from one to the other, and that no lines of force can pass from one to the other through solid insulation material. In parallel with the loss-free capacity *1-2* are the two capacities *1-3* and *3-2* in series which are not free of loss (three-point capacity, see *b*). The auxiliary capacities are rendered harmless in the measurement by a method discussed later.

so that they are insulated from each other. To do this it is unavoidable that some kind or other of solid insulation material must be introduced into the electric field between the electrodes or connections, and in this solid dielectric losses again occur, much larger than those in air, not only due to the smaller insulation resistance of the solid substance, but also due to the polarisation always occurring in it.

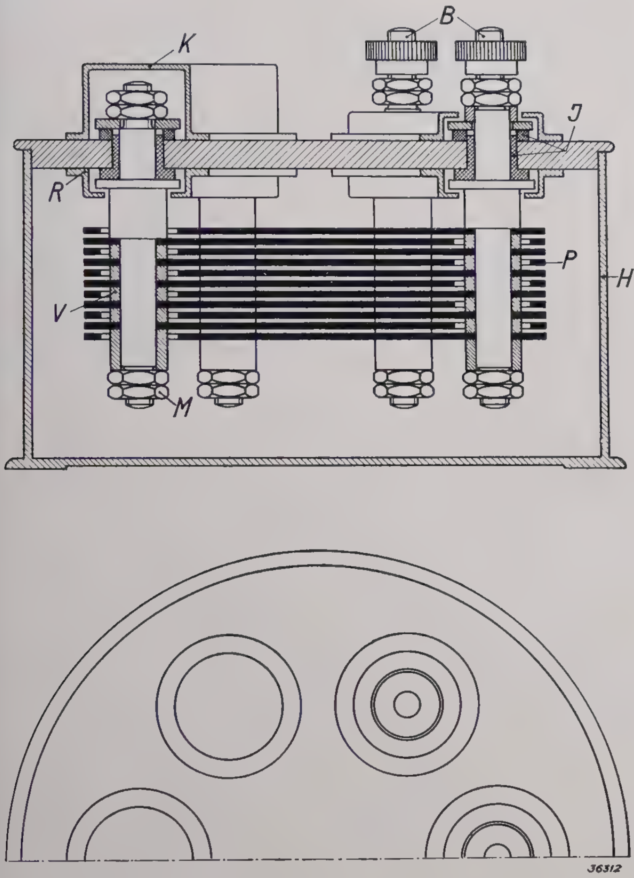

Fig. 4. Construction of the standard condenser. Three pillars, placed in a triangle, support one set of plates *P*, three other pillars in a triangle rotated 60° with respect to the first support the second set of plates lying between the first. This is shown clearly in the detail sketch. The plates are held the correct distance apart by means of distance pieces *V* which are secured by the nuts *M*. The pillars are fastened into the metal housing *H* (the third electrode, see fig. 3) with an insulation of high-frequency "Philite". The metal guard rings *R* and caps should be noticed, which shield the insulation and the free ends of the pillars in order to prevent an undesired "leakage of lines of force". *B* are the two binding posts.

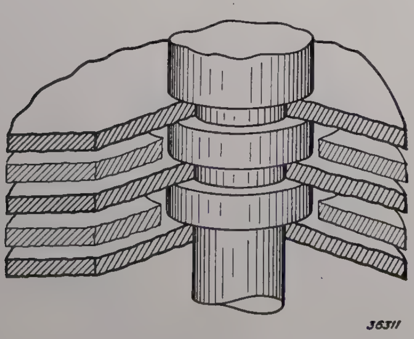

Fig. 4*a*. Detail of the construction.

These losses can now be rendered harmless by application of the principle illustrated in *fig. 3a*. The two electrodes are assembled with mutual insulation in, and entirely surrounded by a metal housing which serves as third electrode, and which can be brought to a certain potential. No leakage current can now flow directly from pole *1* to pole *2*, and with suitable construction no lines of force running from *1* to *2* can anywhere encounter a solid substance. The direct capacity between the points *1* and *2* is therefore practically entirely free of losses. It is, however, true that we have gained two additional capacities in parallel with *1-2* (fig. 3*b*), namely between *1* and *3* and between *2* and *3*, which may have quite considerable losses. By means of a special bridge connection which will be discussed later, it can, however, be brought about that only the capacity between *1* and *2* of the three-point capacity *1-2-3* makes itself felt in the measurement.

Fig. 4 shows how the principle described is realized. The condenser plates are borne by two sets of pillars which are fastened in the upper plate of the metal housing with an insulation of high-frequency "Philite". It is essential that metal guard rings and caps should be placed on the plates to which the pillars are fastened as well as on the free ends of the pillars and connected to the housing, in order to avoid as far as possible the danger that lines of force between poles *1* and *2* should cut the "Philite".

When we have rendered the insulation material harmless in this way, there still remains a spot where the electric field passes through a solid dielectric, namely on the condenser plates themselves. In the case of the metals usually used for this purpose the surface is never entirely clean, but always bears a semi-conducting film of oxide. This could only be avoided by using a precious metal such as gold. If, however, massive plates are used, this would be much too expensive, since in order to avoid any microphone effect (the capacity must be fixed within 10^{-4} per cent) the plates must be very thick, and in order to obtain the desired capacities a fairly large surface of plate is required. Gilding the plates was also found to be impossible, since alloys were formed with the underlying metal. The evil of the oxide film could, however, be very much limited when the plates were made of aluminium or covered with a layer of aluminium. An oxide film is then formed, it is true, but it is much thinner than with other metals (several microns), and thanks to its poor conductivity it has only a very small phase displacement.

Determination of the remaining losses of the standard condensers

What will be the error in the determination of tan δ in the measurements if we neglect the losses of the standard condenser itself? In other words how large may the losses be which the standard condenser still has in spite of the preventive measures described above?

The phase displacement due to the losses in the metals, *i.e.* the resistance of pillars, plates and contacts (as well as eddy current losses occurring at higher frequencies) still remains below the accuracy of measurement attained with our bridge connections, even at the highest frequency with which we are concerned (8 000 c/s), as calculations show. The same is true of the estimated losses in the air. We have therefore still to consider only *a*) the losses in the oxide films which cause a phase displacement δ_a and *b*) the remaining losses in the solid insulation material which lead to a phase displacement δ_b. The total phase displacement of the condenser is then

$$\delta = \delta_a + \delta_b.$$

(Since we are concerned with very small angles we may here consider the angle itself instead of its tangent).

In order to determine δ we have constructed different standard condensers — as has already been stated, a number of them were needed —, whose construction differed in such a way that either δ_a or δ_b was changed in a known manner. The differences in δ for each pair of standards can be measured directly, according to equation (3), and in this way sufficient equations were obtained to calculate all the values of δ_a and δ_b separately.

In practice this was done in the following way. Two standard condensers C_1 and C_2 were made each of 100 pF [1]), furthermore a condenser C_3 of 200 pF. In the case of C_1 and C_3 the distance between the plates was 1 mm. In C_2 it was 5 mm (C_2 therefore also had a surface 5 times as large as C_1) while in other respects the construction of all was the same. It is obvious that if the oxide film is much thinner than the distance between the plates, δ_a is inversely proportional to the latter. Thus for the condenser C_1 δ_a is 5 times as large as for C_2, while for both condensers δ_b must have the same value, since the condensers are identical in all details which can influence δ_b. By measuring the difference in δ for C_1 and C_2, δ_a can be calculated for both condensers. The measurement was carried out at 1 000 c/s and at 8 000 c/s, and gave for both frequencies a value $\delta_a \approx 5 \times 10^{-8}$ for C_1 and $\delta_a \approx 10^{-8}$ for C_2. The value of δ_a for C_3 is the same as for C_1 since the two condensers have the same distance between the plates.

C_1 and C_2 were now connected in parallel giving a capacity of 200 pF which could be compared in the bridge with C_3. The parallel conductance of type b) in C_3 is equal to that in C_1 or C_2, and in $C_1 + C_2$ it is twice as much. By measurement of the difference in δ for C_3 and $C_1 + C_2$, δ_b could also be calculated for all three, since δ_a is already known for all three. The value of δ_b was found to lie below the limit of accuracy of our bridge.

In order to cover the whole desired capacity region a standard of 1 000 pF had also to be constructed; by a measurement in different steps the above result could be tested and confirmed with this standard.

The total phase displacement of the standard condenser obtained amounts, according to these results, to several times 10^{-8}, and may therefore certainly be neglected in the measurements which we had in mind, where an accuracy of 10^{-6} was required.

Connections of the measuring bridge

From the description of the standard condenser it has already appeared that our measurements would be entirely wrong if we introduced the standard condenser into the bridge connections of fig. 2 in place of C_N without further precautions. If this were done, the two auxiliary capacities of the standard condenser connected in series would then be in parallel with C_N (see fig. 3*b*). Moreover, it is necessary to surround the whole bridge by an earthed screen in order to avoid disturbing influences from the outside (hand effect). By this means considerable additional parasitic impedances between points on the bridge and earth are introduced. If the housing of the normal condenser is also earthed, it is easy to divide the disturbing parasitic impedances into two groups: those between bridge points and earth, and those between points on an arm of the bridge and earth. We shall consider these in succession.

Parasitic impedances between corners of the bridge and earth

The error which is caused by parasitic impedances between corners of the bridge and earth may be understood as follows. The bridge is so balanced that the detection apparatus is silent, and from this we conclude that the current in the bridge arm from *1* to *2* is the same as in the arm from *2* to *3*, from which Wheatstone's condition follows. The conclusion as to the equality of the currents from *1* to *2* and from *2* to *3*, however, is only justified when from point *2* no appreciable current flows to earth, *i.e.* when there is no parasitic impedance between *2* and earth.

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[1]) The abreviation *p* (pico) is equivalent to μμ and means a factor 10^{-12}; 1 pF is thus 1 μμF = 10^{-12} F.

The parasitic impedance actually present can now be rendered harmless by means of an auxiliary bridge connection (earth connection according to Wagner) which is given in *fig. 5*. The corner *5* of the auxiliary bridge arms is earthed, and the impedances Z_{12}, Z_{23}, Z_{34}, Z_{41} of the four arms of the main bridge, as well as the impedances Z_1, Z_3 of the auxiliary bridge are so balanced that the detector gives no signal when connected between *2* and *5* and between *4* and *5*. In this case *2* and *4* are at earth potential and therefore no current can flow through parasitic impedances between *2* or *4* and earth. The current from *1* to *2* is thus indeed equal to that from *2* to *3*, and that from *1* to *4* is equal to that from *4* to *3*, so that Wheatstone's equation is valid

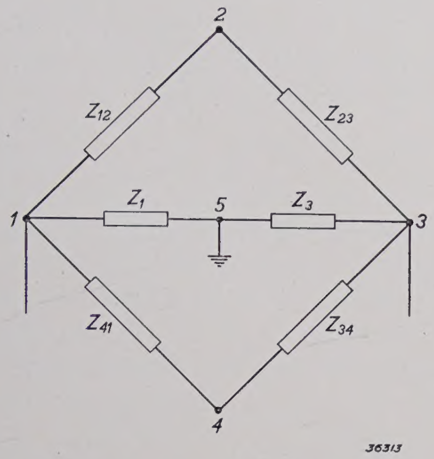


Fig. 5. Auxiliary bridge connection (earth connection according to Wagner). By balancing the (partially parasitic) impedances Z_1 and Z_3 all the parasitic impedances between the corners of the bridge and earth are rendered harmless.

$$Z_{12}Z_{34} = Z_{23}\,Z_{41} \quad . \; . \; . \; . \; . \quad (4)$$

At the same time the corresponding condition for the auxiliary bridge:

$$Z_{41}Z_3 = Z_{34}\,Z_1 \quad . \; . \; . \; . \; . \; . \quad (5)$$

must also be satisfied, where the parasitic impedances between *1* and *3* and earth are taken into account in Z_1 and Z_3, respectively. The parasitic impedances of all four corners of the main bridge are rendered harmless in this way.

The method consists practically in the fact that we introduce into the equal-armed bridge of fig. 2 the arms *5-1* and *5-3*, each of which consists of a variable condenser with a variable resistance in parallel with it. By adjustment of the condenser and the resistance we can make the total impedance between *5* and *1* (*i.e.* including the parasitic impedance) equal to that between *5* and *3*.

The drift of the parasitic impedances

A new difficulty now appears: the parasitic impedances are not constant. They are formed by all kinds of capacities, especially those between connections, transformers, etc. and earth, all of which have large phase displacements. These phase displacements, and with them the effective capacities also, depend very much upon the temperature and other circumstances. The result is that the adjustment of the auxiliary bridge would have to be continually controlled, or, since this is scarcely possible in practice, that a certain error Δ is continually made in the balancing of the main bridge. By this we mean that the detection apparatus is silent when it is not true that

$$Z_{12}Z_{34} - Z_{23}Z_{41} = 0$$

(equation (4)), but when

$$Z_{12}Z_{34} - Z_{23}Z_{41} = \Delta.$$

Let us now consider the bridge in fig. 5 once more, including the (parasitic) impedances Z_2 and Z_4 between *2* and *4*, respectively, and earth (see *fig. 6*).

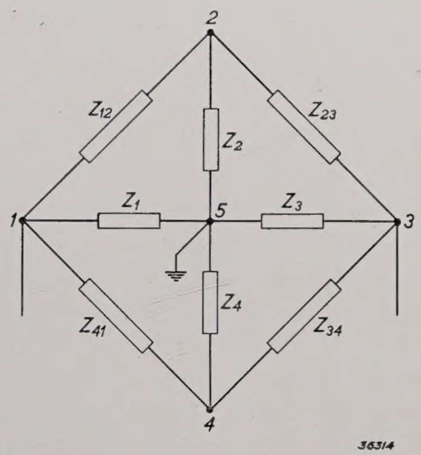


Fig. 6. Entirely symmetrical bridge with two auxiliary bridge connections. By balancing the (originally exclusively parasitic) impedances Z_2 and Z_4, the influence of the drift of all the parasitic impedances is limited.

The general proposition can then be proved: that from the fact that a detector introduced between *2* and *4* is silent, equation (4) may be considered to hold, if

$$(Z_1Z_{34} - Z_3Z_{41})\,(Z_4Z_{12} - Z_2Z_{41}) = 0 \quad . \quad (6)$$

That is, all the parasitic impedances of the corners of the bridge are harmless if we make either

$$Z_1Z_{34} - Z_3Z_{41} = 0 \quad . \; . \; . \; . \; . \quad (5a)$$

(this was the earth connection according to Wagner described above) or

$$Z_4 Z_{12} - Z_2 Z_{41} = 0 \quad . \quad . \quad . \quad . \quad . \quad (5b)$$

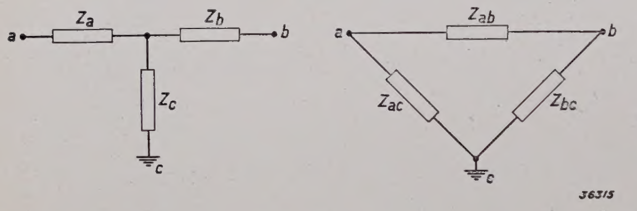
If, however, in the adjustment of the first auxiliary bridge (Z_1, Z_3) an error Δ_1 was made, and in that of the second auxiliary bridge (Z_2, Z_4) an error Δ_2, so that instead of (*5a*, *b*) the following is valid:

$$Z_1 Z_{34} - Z_3 Z_{41} = \Delta_1,$$
$$Z_4 Z_{12} - Z_2 Z_{41} = \Delta_2,$$

then the error in the main bridge is found to be proportional to these two errors:

$$\Delta \sim \Delta_1 \cdot \Delta_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In the case of the simple earth connection according to Wagner first described, we attempted to make $\Delta_1 = 0$, but disregarded the error Δ_2. The result, since Δ_2 may have an appreciable value, was that there was immediately a considerable error Δ at the least deviation of Δ_1. The remedy is now obvious. While the conditions (4) and (5a) for the balancing of the bridge remain valid, Z_2 and Z_4 will at the same time be so chosen that equation (5b) is satisfied. In the bridge which we constructed Z_2 was a fairly large parasitic capacity. By connecting in parallel with the parasitic impedance between 4 and earth, another variable self-induction with a variable resistance in series, Z_4 could be so adjusted that $Z_4 Z_{12} - Z_2 Z_{41} = \Delta_2 = 0$. Upon a drift in the impedance both factors which cause an error in the adjustment of the main bridge (equation (7)), now remain sufficiently small.



a) *b*)

Fig. 7. The parasitic impedance Z_c between a point on the arm of the bridge *a-b* and earth is equivalent to two parasitic impedances between the corners of the bridge *a* and *b* and earth, in which the effective impedance of the bridge arm Z_{ab} is not equal to $Z_a + Z_b$.

Parasitic impedances between points on the bridge arms and earth

In the case of the second group of parasitic impedances, those between points on an arm of the bridge and earth, we must pay particular attention to the auxiliary capacities of the standard condenser. In the bridge given in fig. 2 (see also fig. 8*a*) the standard condenser C_N was situated in one arm in series with the resistance R, from whose adjustment the phase displacement δ_X was calculated. Between the condenser pole connected to R and the earthed condenser housing there exists a considerable parasitic impedance. What results may this have on the measurements?

The configuration is of the type represented in *fig. 7a*. The impedances Z_a (*i.e.* the loss-free capacity C_N) and Z_b (*i.e.* the resistance R) together form the arm of the bridge, Z_c is the parasitic impedance. The star connection of fig. 7*a* is equivalent to a triangle which is represented in fig. 7*b*. The impedance Z_{ab} in the latter is

$$Z_{ab} = Z_a + Z_b + \frac{Z_a Z_b}{Z_c} \quad . \quad . \quad . \quad . \quad (8)$$

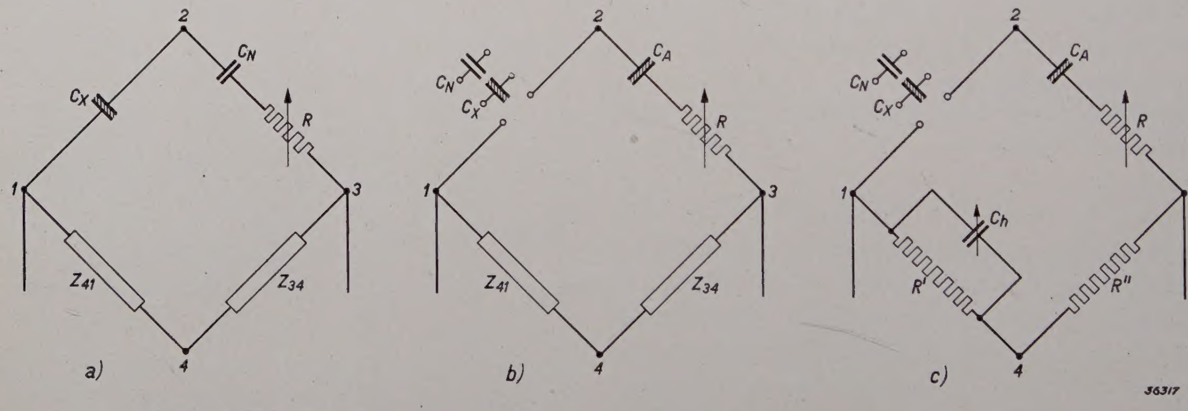


Fig. 8.*a*) Equal-armed bridge, as fig. 2.
b) Equal-armed bridge with application of a substitution method. As comparison condenser C_A a condenser is taken of about the same quality as the condenser C_X being examined. Since C_A has losses and C_N has none, negative values of the series resistance R would be necessary to bring the bridge into equilibrium.
c) By taking for Z_{41} and Z_{34} two equal resistances R' and R'' and introducing a variable auxiliary condenser C_h in parallel with R' the bridge can be brought into equilibrium with positive values of R.

The impedances Z_{ab} and Z_{bc} may be considered as parasitic impedances between corners of the bridge and earth, and thus play no part in the application of the auxiliary bridge connections. The effective impedance of the bridge arm is given by (8), while neglecting Z_c, we had considered the sum $Z_a + Z_b$ as the effective impedance. We thus make an error

$$\Delta Z_{ab} = Z_a Z_b / Z_c.$$

In our standard condenser of 100 pF the auxiliary capacity $C_p \approx 80$ pF, therefore

$$\Delta Z_{ab} = R \cdot C_p / C_N = 0.8\ R.$$

Thus, while in our measurements of loss R must be determined accurately to within fractions of a hundredth of one per cent, we must here introduce a correction of 80 per cent! It is unnecessary to state that this would make the measurements very difficult and unreliable, considering the lack of constancy of C_p.

We have avoided this difficulty by the application of a substitution method, see *fig. 8b*. In the arm *1-2* the condenser being examined C_X and the standard condenser C_N are successively introduced, while the adjustable resistance R, which is connected in the arm *2-3* in series with an arbitrary comparison condenser C_A is in each case so adjusted that the bridge reaches an equilibrium. The difference between the two values of R gives the effective series resistance, and thus the phase displacement of C_X. The difficulty mentioned — that of the parasitic impedance between the condenser pole at the middle of the bridge arm and earth — does not occur at the point of connection between C_A and R, if we take for C_A a variable air condenser of the ordinary type which can be purchased (C_A must be of about the same quality as C_X). With these condensers, which have no third electrode, the housing is connected to one electrode, so that the other electrode is entirely surrounded by it, see *fig. 9*. By connecting the housing to point *2* of the bridge, and also by surrounding the resistance R with a screen which is connected to point *3* of the bridge and which surrounds the connecting wire between C_A and R up to close to C_A, it is possible to make the remaining capacity between the dangerous connection point and earth only very small.

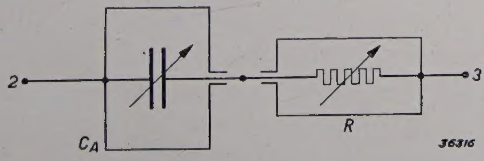


Fig. 9. If C_A is an air condenser of the ordinary type (two-point capacity), the parasitic capacity between the connection point of C_A and R and earth can easily be made very small.

In the practical execution of the substitution method it must still be kept in mind that since C_A has losses and C_N has none, a negative resistance R would be necessary to bring the equal-armed bridge into equilibrium. The bridge, however, need not be equal-armed. If we take for Z_{41} and Z_{34} two equal resistances R' and R'', and if we then connect a variable auxiliary condenser C_h (fig. 8*c*) in parallel with R', by regulation of this condenser we can bring the bridge so far out of equilibrium that a positive value of R is required to restore the equilibrium not only upon switching in C_X but also upon switching in C_N [2]).

In order to be able to measure a phase displacement $\tan \delta = 3 \times 10^{-8}$ with a capacity $X_X = 100$ pF and at a frequency of 8 000 c/s, R must be able to be adjusted with an accuracy of 0.005 ohm.

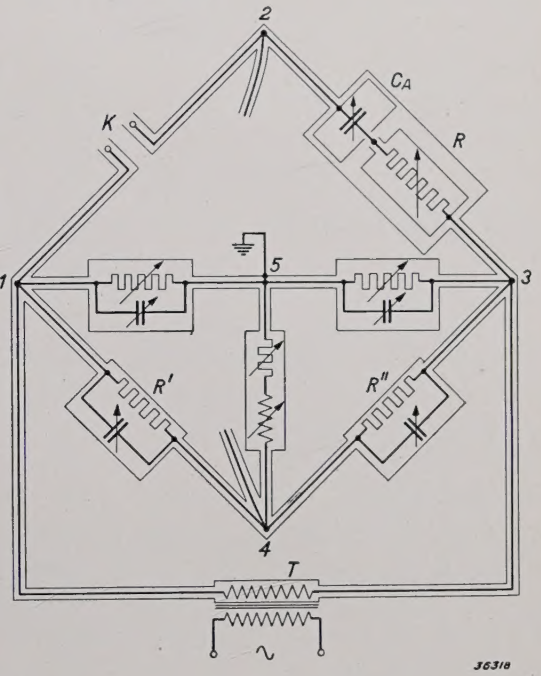


Fig. 10. Complete connections for the bridge used by us. In order to avoid disturbances from the outside the bridge is entirely surrounded by an earthed shield. At the terminals K, C_X or C_N is connected. The elements used to realize the impedances Z_1, Z_3 and Z_4 of the auxiliary bridge arms (see fig. 6) may be seen. These impedances must be balanced. A small variable auxiliary condenser is connected in parallel with R'' as well as with R', in order to facilitate the balancing. The bridge is supplied from the shielded transformer T. The detector, which is not shown, is connected successively at the points *2-4*, *2-5* and *4-5*.

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[2]) Actually one ought also to be able to omit the resistance R entirely, and to bring the bridge exactly into equilibrium each time only by regulating C_h. The difference between the two adjustments of C_h necessary upon switching in C_X and C_N, respectively, would then give $\tan \delta_X$. This is the principle of Schering's bridge. We have not used this method because of the fact that extremely heavy requirements would be made of the condenser C_h: it would have to assume capacities up to 20 pF which would have to be accurately adjustable within 0.0005 pF!

at a total value of $R = 200$ ohms. Such a resistance can very well be realized.

In *fig. 10* we give finally the complete connections of the bridge as it has been developed on the basis of the above considerations. In the text below the figure certain details are explained.

Accuracy of measurement

As to the accuracy of measurement obtained, it is limited in principle by the accuracy with which the bridge can be adjusted to silencing the detector. The accuracy of adjustment is higher, the higher the voltage applied to the bridge and the greater the amplification applied in the detection apparatus. The voltage cannot be raised indefinitely because of the limited permissible load on the resistances R' and R'' (fig. 10). A limit is set to the effective amplification of the signal to be detected by the noise. This limit has been set as high as possible by sifting out a very narrow frequency range on both sides of the measuring frequency with the help of sharp cut-off filters, so that only this small section of the whole noise spectrum can become audible in the telephone as a tone fluctuating in intensity. When this fluctuating tone is heard, further amplification serves no useful purpose. The smallest measurable impedance change in the bridge is that at which the signal caused by it is just audible above the noise tone; it is found that in this way a change in tan δ of 3×10^{-8} can just be observed. Since the systematic errors in the substitution method used, as has been shown in different tests, are certainly smaller than the adjustment error mentioned of 3×10^{-8}, this is also the limit of the absolute accuracy with which we can measure a phase displacement with the apparatus described.

ABSTRACTS OF RECENT SCIENTIFIC FUBLICATIONS OF THE N.V. PHILIPS' GLOEILAMPENFABRIEKEN

1504: W. de Groot: Miscellaneous observations on the rise and decay of the luminescence of various phosphors (Physica **7**, 432-446, May 1940).

With the apparatus described in **No. 1425** the rise and decay of the photoluminescence of different crystals is further studied. Crystals of ZnS-Ag, ZnS-Cu, ZnSCdS-Ag, ZnSMnS, $CaWO_4$-Sm, Zn_2SiO_4-Mn_2SiO_4 were investigated at different temperatures and different wave lengths of the irradiating light. The effective diameter of the luminescent centres for excitation by electrons, calculated from the rise of the luminescence, for the last mentioned phosphor agrees very well with the value derived from the saturation of the luminescence from data of Nottingham, while, with the help of data also from this author, a very acceptable value is found for the number of centres per unit of volume. In the quantitative interpretation of the experimentally obtained curves for the variation of the luminescence, the influence of various causes of deformation must be taken into account, among others, for instance, the weakening of the irradiating light upon penetration into the crystal. The effect of this is calculated. Furthermore with the help of an ultramicroscope the penetration of monochromatic light beams of different wave lengths into a single crystal of ZnSCdS-Ag is investigated.

1505: J. L. Snoek: On the dispersion of the initial permeability (Physica **7**, 515-518, June 1940). (Original in German).

Experiments have shown that a sample of carefully purified iron, well annealed and at frequencies of 1 c/s has an initial permeability which is many times greater than the initial permeability found from measurements of the skin effect at frequencies of 10^6 to 10^8 c/s. The present theory offers no explanation of this discrepancy.